

EQUILIBRIA UNDER GENERALIZED MODEL OF SPATIAL COMPETITION

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I Introduction

The purpose of this paper is to investigate the equilibria under the generalized model of spatial competition. The spatial competitive equilibrium has been analysed by the two different models of spatial competition, the Löschian model and the Greenhut and Ohta model¹⁾²⁾. Both models were originally designed for analysing the market shapes. These models represent the typical ways of spatial competition. The models thus are used to analyse the spatial economy and the spatial competitive equilibrium.

Considering the equilibria under these models, we notice that the equilibria are strongly tied to the typical competition way used in each model and the equilibria do not generally represent the spatial competitive equilibria.

We shall consider the spatial competition from the aspect of the firm's attitude of competition and construct the model of spatial competition. This model will contain the Löschian and the Greenhut = Ohta model as special cases in this model. We shall analyse the equilibria under this generalized model and show the mill prices and the market sizes of firms in spatial competitive equilibria.

In the next two sections we explain basic assumptions and derive the profit functions of firms for three kinds of market shapes. In the section IV we consider the Löschian and the Greenhut = Ohta model and show the values of equilibrium mill prices and market sizes of firms in both models. The section V shows the method of constructing the generalized model of spatial competition and derives the equilibrium values of mill prices and market sizes.

The results of the investigations are summarized in the section VI.

II Assumptions

The argument is subject to the following assumptions.

- 1) Consumers are uniformly and continuously distributed in a plain.
- 2) Each consumer has the same demand curve;

$$q = a - g(p + wu) \quad (1)$$

where q is the demand per consumer, p is the mill price, u is the distance from the firm to a consumer, w is the transportation cost per unit mile and a, g are positive parameters (In the following sections we assume w and g are 1 respectively).

- 3) Every consumer should access to goods.
- 4) Consumers bear transportation cost.
- 5) Each firm has the same cost function;

$$TC = F + KQ \quad (2)$$

where TC is total cost, F is fixed cost, K is marginal cost, Q is total sales.

- 6) Firms enter into the plain without restrictions.

The market shapes which satisfy the assumption 3) are triangles, squares and hexagons. Then we shall consider the equilibrium prices and market sizes in relation to the three kinds of market shapes.

III Profit functions for the three kinds of market shapes.

We first take the profit functions for each market shape. The profit of a firm is given such that:

$$Y = Q(P - K) - F \quad (3)$$

where Y stands for profit of a firm. To obtain the profit of a firm, we have to know the total sales of the firm. In general, for any regular polygonal market area of the inscribed circle radius U , total sales Q of a firm will be:

$$Q = 2\rho \int_0^{\pi/\rho} \int_0^{U/\cos\theta} [a - (p + u)] u du d\theta \quad (4)$$

where ρ is the number of sides of regular polygon, θ is the angle in any triangle which connects the firm in the center of the polygon to a set of consumer sites located on the vertex and the closest point on the perimeter of the market area. profits for each market shape, triangular (t), square (s) and hex-

agonal (h) can be derived from equations (3) and (4). and are represented as:

$$\begin{aligned} Y_t &= 6U_t^2(\sqrt{3}/2 \cdot a - \sqrt{3}/2 \cdot P_t - 0.7969U_t)(P_t - K) - F(5_t) \\ Y_s &= 8U_s^2(1/2 \cdot a - 1/2 \cdot P_s - 0.3848U_s)(P_s - K) - F(5_s) \quad (5i) \\ Y_h &= 12U_h^2(1/2\sqrt{3} \cdot a - 1/2\sqrt{3} \cdot P_h - 0.2027U_h)(P_h - K) - F(5_h) \end{aligned}$$

IV Equilibrium under the Löschian and the Greenhut=Ohta model

1 Equilibrium under the Löschian model

In the Löschian model it is assumed that locations and market areas of firms are given, the mill prices are variables; firms determine their mill prices to maximize their profits in the given market areas.

To attain a spatial competitive equilibrium, the two conditions must be satisfied, A) one is that of profit maximization under free entry. B) the other is that of equilibrium under free entry. Since in the Löschian model U is given and P is variable, condition A) is obtained by taking the partial derivatives of equations (5i) with respect to P_i ($i=t, s, h$) and setting them at zero

$$\frac{\partial Y_i}{\partial P_i} = 0 \quad i=t, s, h \quad (6i)$$

the latter condition is

$$Y_i = 0 \quad i=t, s, h \quad (7i)$$

The U_i and P_i ($i=t, s, h$) satisfying the both conditions simultaneously are equilibrium values. They thus are obtained by solving simultaneous equations (6i) and (7i). For the sake of the simplicity without losing generality, we assume marginal cost is zero, and solve the simultaneous equations³⁾. Table 1 shows the equilibrium price P_i^* and the inscribed circle radius U_i^* when the fixed cost is $0.05a^4$.

Table 1. Equilibrium P and U for the three kinds of shapes under the Löschian model

Triangle		Square		Hexagon	
P^*	U^*	P^*	U^*	P^*	U^*
0.3818a	0.2569a	0.3900a	0.2870a	0.3925a	0.3061a

$$F = 0.05a^4$$

2 Equilibrium under the Greenhut=Ohta model

The Greenhut=Ohta model is founded on the supposition that mill prices,

locations and market sizes of firms are variables, the delivered prices at the boundaries are given; firms determine the mill prices to maximize the profits according to the delivered prices at the boundaries. Once the firms determine the mill prices, the locations and market sizes are in turn given uniquely. In other words, each firm can enlarge, moving its location, the market area by lowering the mill price.

In this model, U is variable and the delivered price at the market boundary ($\bar{P}$) is given, thus equations (5i) must be rewritten as function of U_i and $\bar{P}_i$; equations (5i) are transformed as follows: We use $\bar{U}_i$ ($i=t, s, h$) to stand for the longest distance from the firm to the perimeter of the market area. Then the mill prices are expressed in terms of $\bar{U}_i$ and $\bar{P}_i$,

$$P_i = \bar{P}_i - \bar{U}_i \quad i=t, s, h \quad (8i)$$

Since $U_t = 1/2 \cdot \bar{U}_t$, $U_s = 1/\sqrt{2} \cdot \bar{U}_s$, $U_h = \sqrt{3}/2 \cdot \bar{U}_h$, P_i are expressed in terms of U_i and $\bar{P}_i$,

$$P_i = \bar{P}_i - r_i U_i \quad (9i)$$

$$r_t = 2, \quad r_s = \sqrt{2}, \quad r_h = 2/\sqrt{3}.$$

Substituting the P_i of (8i) into equations (5i) gives the profit, Y_i , as a function of U_i and $\bar{P}_i$;

$$Y_i = \alpha_{1i} U_i^4 + \alpha_{2i} U_i^3 + \alpha_{3i} U_i^2 - F \quad (10i)$$

$$\alpha_{1t} = -11.2218, \quad \alpha_{2t} = 6(2.6672 \bar{P}_t - \sqrt{3}a - 0.9352k), \quad \alpha_{3t} = 3\sqrt{3}(a\bar{P}_t - ak - \bar{P}_t^2 + \bar{P}_t k)$$

$$\alpha_{1s} = -3.6465, \quad \alpha_{2s} = (1.0294 \bar{P}_s - a/\sqrt{2} - 0.3223k), \quad \alpha_{3s} = 4(a\bar{P}_s - ak - \bar{P}_s^2 + \bar{P}_s k)$$

$$\alpha_{1h} = -1.8101, \quad \alpha_{2h} = 12(0.4639 \bar{P}_h - a/3 - 0.1306k), \quad \alpha_{3h} = 2\sqrt{3}(a\bar{P}_h - ak - \bar{P}_h^2 + \bar{P}_h k)$$

For a equilibrium, the two conditions A) and B) shown above section are to be satisfied simultaneously. Since in the Greenhut = Ohta model U is variable and $\bar{P}$ is parameter, the condition A) is obtained by taking the partial derivatives of equations (10i) with respect to U_i and setting them at zero:

$$\frac{\partial Y_i}{\partial U_i} \quad i=t, s, h \quad (11i)$$

the condition B) is

$$Y_i = 0 \quad i = t, s, h \quad (12i)$$

Equilibrium values of $\bar{P}$ and U_i under this model are obtained by solving the simultaneous equations (11i) and (12i). P^*i and U^*i are shown at table 2 where F and K are assumed $0.05a^4$ and 0 respectively. Equilibrium P_i are obtained by using equation (9i).

Table 2. Equilibrium P and U for the three kinds of shapes under the Greenhut=Ohta model

Triangle		Square		Hexagon	
P^*	U^*	P^*	U^*	P^*	U^*
0.2227a	0.2913a	0.2001a	0.3409a	0.1874a	0.3744a

$$F = 0.05a^4$$

A difference between the Löschian and the Greenhut=Ohta model is as follows: In the former U_i ($i=t, s, h$) are given and P_i are variable, hence the change of P_i does not affect U_i , that is, $dU_i/dP_i = 0$: In the latter model, $\bar{P}_i$ are given, P_i and U_i are variable, hence the change of P_i , moving the firm's location, affects U_i directly, that is, $dU_i/dP_i = 1/r_i$. The differences in values of dU_i/dP_i cause equilibrium values of P_i and U_i to be different in the Löschian and the Greenhut=Ohta model.

V Equilibria under generalized model of spatial competition

The effects of change of P_i on U_i , dU_i/dP_i , are not restricted to $1/r_i$ or 0 , they might any other values according to the situations surrounding firms. When dU_i/dP_i are not $1/r_i$ or 0 , the resulting equilibria are different from those of the Löschian or the Greenhut=Ohta model. To analyse such equilibria, we construct, basing on a view point different from the Löschian and the Greenhut=Ohta model, another model of spatial competition.

We consider the way of spatial competition from the aspect of the firms' competition attitudes which are shown in the Hotelling and the Smithies model⁽⁴⁾⁽⁵⁾. Suppose that locations of the firms are parametrically given, firms' mill prices and market sizes are variables; firms charge their mill prices to maximize their profits, conjecturing how the rivals that are farthest of rivals surrounding the firms react to the mill prices. Once the mill prices are determined, market sizes are uniquely given. This model would be named Inter-

dependence model.

In this model dU_i/dP_i ($i = t, s, h$) depend on the firms' conjecturing of price reaction. Let P' represent price of the rivals, dU_i/dP_i are determined by dp'/dp . When firm conjectures the price reaction of the rivals that any change in its price will be exactly matched by the rivals, $dP'/dP = 1$, the firm does not expect a decrease in its price to lead to enlarge its market area, $dU_i/dP_i = 0$. This case is consistent with that of the Löschian model. When the firm conjectures that a one-unit decrease in its price will lead to a one-unit increase in the rivals' prices, $dP'/dP = -1$, the firm expects a decrease in its price to lead to enlarge market length by one-unit in direction to the rivals, $d\bar{U}_i/dP_i = 1$. This case is consistent with that of the Greenhut = Ohta. The value of dU_i/dP_i to the each value of dP'/dP , except $dP'/dP = 1$, depends on the market shape. It is derived as follows; $dU_t/dP_t = [(dP'/dP - 1)/2]/2$, and $dU_s/dP_s = [(dP'/dP - 1)/2]/\sqrt{2}$ and $dU_h/dP_h = [(dp'/dp - 1)2] \cdot \sqrt{3}/2$. In the following considerations, we select the three values of price reaction (dP'/dP), $dP'/dP = 1, -1$ and $1/2$, and proceed with analysis for the three values of dP'/dP .

To attain equilibrium, the two equilibrium conditions A) and B) are to be satisfied simultaneously. The condition A) is obtained by taking the partial derivatives of equation (5i) with respect to P_i ($i = t, s, h$), setting them at zero,

$$\frac{\partial Y_i}{\partial P_i} = \left[(a_i U_i - \beta_i U_i P_i - \gamma_i U_i^2)(P_i - K) \right] \frac{dU_i}{dP_i} + (1/2 \cdot a_i U_i^2 - \beta_i U_i^2 P_i - 1/3 \gamma_i U_i^3 + \beta_i K U_i) = 0 \quad (13i)$$

$$\left\{ \begin{array}{l} \alpha_t = \sqrt{3} \cdot a, \quad \beta_t = \sqrt{3}, \quad \gamma_t = 2.3907, \quad \alpha_s = 2 \cdot a, \quad \beta_s = 2, \quad \gamma_s = 2.3088, \\ \alpha_h = 1/\sqrt{3} \cdot a, \quad \beta_h = 1/\sqrt{3}, \quad \gamma_h = 0.6081 \quad i = t, s, h \end{array} \right.$$

the condition B) is shown by equation (7i).

Substituting dU_i/dP_i with the values corresponding to the values of $dP'/dP, 1, -1$ and $1/2$, and solving equations (13i) and (7i) simultaneously, we obtain the equilibrium price P^* and U^* in each value of price reaction. Table 3 shows P^* and U^* in each value of price reaction. In table marginal cost is assumed 0 and fixed cost is changed from $0.01a^4$ to $0.09a^4$ at intervals of $0.02a^4$.

Figure 1 and 2 show equilibrium P and U for square market in each value of dP'/dP in order to show the effect of dP'/dP on the equilibrium P_i and U_i . In figure 1 and 2 fixed cost is changed from $0.01a^4$ to $0.09a^4$ at intervals of

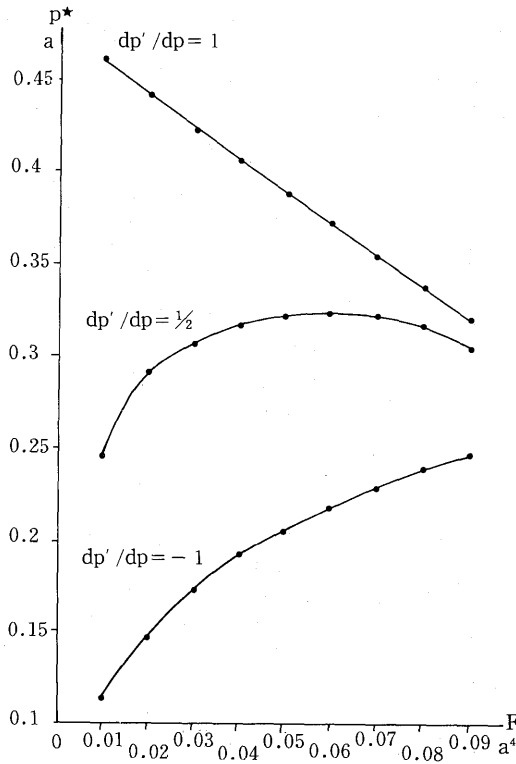
Table 3. Equilibrium P and U for the three kinds of shapes

fixed costs	dp'/dp	Triangle		Square		Hexagon	
		P*	U*	P*	U*	P*	U*
0.01a ⁴	1	0.4557a	0.0963a	0.4580a	0.1092a	0.4589a	0.1171a
	-1	0.1287a	0.1421a	0.1127a	0.1713a	0.1042a	0.1907a
	1/2	0.2673a	0.1065a	0.2419a	0.1250a	0.2267a	0.1371a
0.03a ⁴	1	0.4160a	0.1827a	0.4208a	0.2058a	0.4227a	0.2202a
	-1	0.1877a	0.2255a	0.1665a	0.2679a	0.1549a	0.2963a
	1/2	0.3239a	0.1883a	0.3054a	0.2155a	0.2927a	0.2333a
0.05a ⁴	1	0.3818a	0.2569a	0.3900a	0.2870a	0.3925a	0.3061a
	-1	0.2227a	0.2913a	0.2001a	0.3409a	0.1874a	0.3744a
	1/2	0.3310a	0.2601a	0.3212a	0.2930a	0.3129a	0.3145a
0.07a ⁴	1	-----	-----	0.3577a	0.3700a	0.3621a	0.3925a
	-1	0.2463a	0.3596a	0.2257a	0.4117a	0.2133a	0.4482a
	1/2	0.3193a	0.3377a	0.3186a	0.3734a	0.3147a	0.3976a
0.09a ⁴	1	-----	-----	0.3191a	0.4701a	0.3267a	0.4933a
	-1	-----	-----	0.2455a	0.4957a	0.2349a	0.5313a
	1/2	-----	-----	0.3011a	0.4717a	0.3028a	0.4958a

dp'/dp : price reaction

0.01a⁴. As shown these figures, as the fixed cost becomes higher, U* s become higher independent of the values of dP'/dP. but the changes of the values of P* depend on the value of dP'/dP. When dP'i/dPi = 1, that is Löschian case, Ps* decreases straight, when dP'/dP = -1, Greenhut=Ohta case, Ps* increases monotonously, when dP'/dP = 1/2, Ps* increases up to a point and in turn decreases. When dP'/dP is in -1 < dP'/dP < 1, the change of Ps* is like that of dP'i/dPi = 1/2. And when dP'i/dPi is relatively high, Ps* becomes high and Us* becomes low relatively. The same things can be said to the triangular and hexagonal market shapes. Comparing the values of equilibrium Pi and Ui shown at table 1, 2 and 3, the values are tied to the fixed cost of 0.05a⁴, it is confirmed clearly that this model contains the Löschian and the Greenhut=Ohta model as special cases.

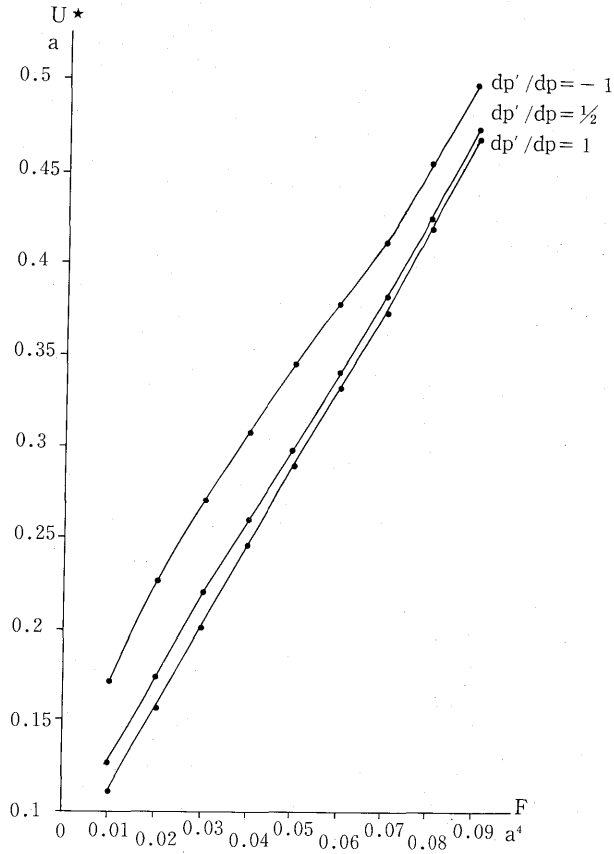
Figure 1. Equilibrium P for Square market area under three values of dp'/dp .



VI Conclusions

The Interdependence model takes into account the widely ranging values of price reaction, the Löschian and the Greenhut = Ohta model are contained in this model as special cases. Therefore, this model could have more generality than the Löschian and the Greenhut = Ohta model in analysing the spatial competitive equilibria. The analyses under the model lead to the following conclusions: When firm conjectures the prices of rivals are close to its own, equilibrium P becomes higher: As the fixed cost is higher, equilibrium U becomes higher, while equilibrium P depends on the value of price reaction, and the differences of these values of equilibrium P and U, based on the difference of price reaction, are reduced gradually.

Figure 2. Equilibrium U for Square market area under three values of dp'/dp .



Notes

1. A. Lösch; Die räumliches Ordnung der Wirtshaft, 1962, Stuttgart.
2. M. Greenthut and H. Ohta; "Spatial configurations and competitive equilibria", Weltwirtschaftliches Archiv, Bd, 109, 1973, Heft , PP. 87-104.
3. For a method of calculation, See k. Suzuki , M. Kaneda and T. Ishikawa ; "Configurations of market areas in spatial competitive equilibria", Annals of the Association of Economic Geographers, Japan, 30, 1984, PP. 1-10.
4. H. Hotelling; "Stabilty in competition", The Economic Journal, vol. 39, 1929, PP. 41-57.

5. A. Smihties; "Optimum Location in Spatial Competition", The Journal of Political Economy, vol. 49, 1941, PP. 423-439.