

RECONSTRUCTION AND APPLICATIONS OF THE CENTRAL PLACE MODEL

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I. INTRODUCTION

The study of central place theory has progressed based on the studies by Christaller and Lösch^{1),2)}, and refinement of the theory and its practical applications have been flourishing^{3),4),5)}. One of these applications is the explanation and analysis of cities and city-size distributions. So far this attempt has not quite been successful. One of the reasons is that the central place theory consists of only hexagonal firm's market areas and lacks a sufficient flexibility for covering various different distribution patterns.

Theoretical bases for the use of hexagonal market areas in the construction of central place theory was first given by Lösch, and have been since supported by other writers. The argument can be summarized as follows. Selecting equilateral triangle, square and hexagon as the shapes of market area, Lösch derives the quantity demanded by consumers in these equi-area market areas. Comparing these quantities demanded, he concludes that the shape of market area in competitive equilibrium becomes hexagon. Later Mills and Lav analyzed the relationship between the shape of market areas and the profits and attempted to determine the shape of market areas in equilibrium⁶⁾. Moreover, Greenhut and Ohta analyzed in a different way the relationship between the shape of market areas and the profits, and approached the problem of the shapes of market areas under the adjustment process of location, that is, the problem of how the locations of the firms in spatial monopoly change due to competition⁷⁾. These two analyses basically reached the same conclusion as Lösch.

Although different analytical methods were used, these three analyses independently showed the following: When the shape of the market areas of firms is hexagon, the number of firms per unit area becomes a maximum. This conclusion provides one of the rationales for the fact that the market area forms a hexagon in spatial competitive equilibrium and therefore hexagonal market areas are used in the model of central place

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theory.

The purpose of this paper is, first, to prove that the shape of the market areas in spatial competitive equilibrium is not necessarily confined to hexagons, but its shape can also be triangles and squares. Second, the present paper attempts to construct the central place model using triangular market areas as well as hexagonal ones and to apply this model to the real data of cities and city-size distributions and thereby to demonstrate that our methodical system owns a superior applicability.

The content of the paper is the following. First, the relationship between the shape of market areas and the profits will be analyzed under Lösch model and Greenhut-Ohta model that frame the most typical spatial competition. This analysis derives the two main results: (1) the shapes of market areas under spatial competitive equilibrium can become, unlike the assertions by Lösch, Greenhut and Ohta, triangles and squares as well as hexagons if selected based on the number of firms per unit area; (2) what shapes the market areas should take depends upon the manner of spatial competition and the level of production costs. Next, a simple but more widely applicable central place model will be constructed by combining both hexagonal and triangular market areas, and an application of this model to the cities and the city-size distribution of the Aichi Prefecture will be attempted.

II. SPATIAL COMPETITION AND EQUILIBRIUM

1. Assumptions

Let us first assume the following.

- 1) Consumers are continuously and uniformly distributed on a plane.
- 2) Each consumer has the following identical demand function:

$$q = a - g(p + wu) \quad (1)$$

where q is the quantity demanded by each consumer, p is the price quoted at factory, u is the distance from a firm to a consumer, w is the rate of transportation cost, a and g are positive constants (assumed 1 respectively hereafter).

- 3) All the consumers can purchase goods.
- 4) Consumers bear transportation costs.
- 5) Each firm has the following identical cost function:

$$TC = F + KQ \quad (2)$$

where TC is total production costs, F is fixed costs, and K is marginal costs and Q is total sales.

6) Firms can enter the market without any restriction.

2. Possible Shapes of Market Areas

First, let us examine possible shapes of market areas that satisfy Assumption 3) above. A circular market area is excluded since it inevitably leaves unsupplied area on a plane. The shapes of the market areas which can completely encompass the same shape of planes can be discovered as follows⁸⁾.

Let us suppose that a plane can be entirely covered by many of the same shape of equilateral polygon. Let that polygon be a regular n -polygon and assume that its n vertices meet at one of vertexes. A vertical angle of the regular n -polygon is

$$(n-2) \pi / n. \quad (3)$$

The condition for m of these vertical angles to become 2π is given by

$$m(1-2/n)=2 \quad (4)$$

From this, $m \cdot n = 2m + 2n$. Adding 4 to both sides of this equation and transforming it, we obtain

$$(m-2)(n-2)=4. \quad (5)$$

The integer solutions for the equation (5) are only three sets of numbers:

$$(m, n) = (4, 4), (3, 6), (6, 3).$$

Thus, the shapes of the market areas that completely cover planes are found to be the following three: a regular triangle, a regular square and a regular hexagon. Analyses on these three types of market areas will be the topics in the following sections.

3. The Profit Functions for the Three Different Shapes of Market Areas

First, we will derive the profit functions for each shape of market area. The profits of the firms are given by

$$Y=Q(P-K)-F \quad (6)$$

where Y is the profits of the firm. In order to calculate the profits, the amount of sales of the firm has to be known. Generally, the amount of sales in any regular n -polygon market area with an inscribed circle of a radius U is given by

$$Q=2\rho \int_0^{\pi/\rho} \int_0^{U/\cos\theta} [a-(p+u)]u du d\theta \quad (7)$$

where ρ is the number of sides of the regular polygon, and θ is the angle formed by the two lines: (1) a line connecting a firm and a vertex of the market area and (2) a line connecting a firm and a middle point of the side. Profits for the market areas of each shape, that is, triangle (t), square (s), and hexagon (h), are derived from the equations (6) and (7) as follows:

$$Y_t = 6U_t^2 (\sqrt{3}/2a - \sqrt{3}/2P_t - 0.7969U_t) (P_t - k) - F \quad (8t)$$

$$Y_s = 8U_s^2 (1/2a - 1/2P_s - 0.3848U_s) (P_s - k) - F \quad (8s)$$

$$Y_h = 12U_h^2 (1/2\sqrt{3}a - 1/2\sqrt{3}P_h - 0.2027U_h) (P_h - k) - F \quad (8h)$$

4. Lösch and Greenhut-Ohta Models and Competitive Equilibrium

In Lösch model, the following is assumed. That is, the location of firms and the size of market areas are given, and the mill price is a variable. Each firm determines the mill price so that it can maximize profits under the given market area.

Now, for spatial competitive equilibrium to be established, the following two conditions have to be satisfied. One is the condition (A) profit maximization under free entry. The other is (B) equilibrium condition under free entry. Since in Lösch model U is given and P is a variable, the condition (A) can be obtained by partially differentiating p_i in the equation (8i) ($i=t, s, h$) and by equating it to zero.

That is,

$$\partial Y_i / \partial P_i = 0 \quad i=t, s, h \quad (9i)$$

The condition (B) can be shown as

$$Y_i = 0 \quad i=t, s, h \quad (10i)$$

The values of U_i and P_i that simultaneously satisfy these two conditions are the equilibrium values U_i^* and P_i^* . Therefore, those values can be calculated by solving the simultaneous equations (9i) and (10i) ⁹⁾.

Next, the Greenhut = Ohta model is based on the following assumption. The mill price, the size of market area and the location are variables, and the delivery price at the furthest point on the boundary of the market area is given. Each firm is assumed to behave to determine the size of market area and therefore the mill price so that it maximizes profits based on that delivery price. In the Greenhut = Ohta model, each firm can, through lowering its mill price, expand the market area while moving its location.

In this model, since U is a variable and the price $\bar{p}$ at the furthest boundary point of the market area is given, the profits function (8i) has to be transformed into the function of U_i and $\bar{P}_i$. This can be done as follows. Let us suppose that $\bar{U}_i$ shows the distance from the firm to the furthest boundary point of the market area. Then, the mill price can be shown by using $\bar{U}_i$ and $\bar{P}_i$ as

$$P_i = \bar{P}_i - \bar{U}_i \quad i=t, s, h \quad (11i)$$

Since $U_t = 1/2\bar{U}_t$, $U_s = 1/\sqrt{2}\bar{U}_s$, and $U_h = \sqrt{3}/2\bar{U}_h$, P_i is shown by using U_i and $\bar{P}_i$ in the following equation:

$$P_i = \bar{P}_i - r_i U_i \quad (12i)$$

$$r_t=2, r_s=\sqrt{2}, r_h=2/\sqrt{3}$$

Putting the equation (12i) into p_i in the equation (8i), the profits (Y') is shown as a function of P_i and U_i in the following:

$$Y'_i = \alpha_{1i}U_i^4 + \alpha_{2i}U_i^3 + \alpha_{3i}U_i^2 - F \quad (13i)$$

$$\alpha_{1t} = -11.2218, \alpha_{2t} = 6(2.6672\bar{P}_t - \sqrt{3}a - 0.9352k),$$

$$\alpha_{3t} = 3\sqrt{3}(a\bar{p}_t - ak - \bar{p}_t^2 + \bar{p}_tk)$$

$$\alpha_{1s} = -3.6465, \alpha_{2s} = 8(1.0294\bar{P}_s - a/\sqrt{2} - 0.3223k),$$

$$\alpha_{3s} = 4(a\bar{p}_s - ak - \bar{p}_s^2 + \bar{p}_sk)$$

$$\alpha_{1h} = -1.8101, \alpha_{2h} = 12(0.4639\bar{P}_h - a/3 - 0.1306k),$$

$$\alpha_{3h} = 2\sqrt{3}(a\bar{p}_h - ak - \bar{p}_h^2 + \bar{p}_hk).$$

In order for an equilibrium to be achieved, the aforementioned two conditions (A) and (B) have to be satisfied simultaneously. Since U is a variable and $\bar{P}$ is given in the Greenhut = Ohta model, the condition (A) is obtained through partially differentiating the equation (13i) by U_i and equating it to zero. That is,

$$\partial Y' / \partial U_i = 0 \quad i=t, s, h \quad (14i)$$

The condition (B) is obtained by putting the equation (13i) to zero.

Thus,

$$Y' = 0 \quad i=t, s, h \quad (15i)$$

The equilibrium values of $\bar{p}_i$ and U_i , $\bar{p}_i^*$ and U_i^* , are obtained by solving the simultaneous equations (14i) and (15i). Incidentally, the equilibrium mill price, p_i^* , can be calculated through the equation (12i). Table 1 shows the equilibrium mill price P_i^* and the radius of the inscribed circle U_i^* in the Lösch and the Greenhut = Ohta models.

In Table 1, for simplicity without loss of generality, the marginal cost is assumed to be zero. The fixed costs are varied by $0.01a^4$ from $0.01a^4$ to $0.1a^4$, and corresponding to each of these fixed costs, each equilibrium value is listed in the table.

III. THE NUMBER OF FIRMS AND THE SHAPES OF MARKET AREAS

The areas of each shape of the market areas in spatial competitive equilibrium can be easily obtained through the radius U_i^* of the inscribed circle in equilibrium. The areas of each shape of market areas in the Lösch and Greenhut = Ohta models are shown in Table 2.

If the areas of market areas are known, it can be readily made clear what shape of market areas can pack the largest number of firms per unit of area. We can tell from

Table 1 . Equilibrium Pi and Ui for the three kinds of Shapes

Fixed Costs	Model	Triangle		Square		Hexagon	
		Pt*	Ut*	Ps*	Us*	Ph*	Uh*
0.01a ⁴	L	0.4557a	0.0963a	0.4580a	0.1092a	0.4589a	0.1171a
	G=O	0.1287a	0.1421a	0.1127a	0.1713a	0.1042a	0.1907a
0.02a ⁴	L	0.4343a	0.1429a	0.4379a	0.1615a	0.4392a	0.1730a
	G=O	0.1633a	0.1885a	0.1441a	0.2255a	0.1336a	0.2502a
0.03a ⁴	L	0.4160a	0.1827a	0.4208a	0.2058a	0.4227a	0.2202a
	G=O	0.1877a	0.2255a	0.1665a	0.2679a	0.1549a	0.2963a
0.04a ⁴	L	0.3988a	0.2200a	0.4050a	0.2469a	0.4047a	0.2638a
	G=O	0.2068a	0.2590a	0.1847a	0.3055a	0.1723a	0.3367a
0.05a ⁴	L	0.3818a	0.2569a	0.3900a	0.2870a	0.3925a	0.3061a
	G=O	0.2227a	0.2913a	0.2001a	0.3409a	0.1874a	0.3744a
0.06a ⁴	L	0.3643a	0.2950a	0.3740a	0.3275a	0.3776a	0.3485a
	G=O	0.2356a	0.3242a	0.2136a	0.3758a	0.2009a	0.4110a
0.07a ⁴	L			0.3577a	0.3700a	0.3621a	0.3925a
	G=O	0.2463a	0.3596a	0.2257a	0.4117a	0.2133a	0.4482a
0.08a ⁴	L			0.3400a	0.4161a	0.3455a	0.4398a
	G=O			0.2363a	0.4547a	0.2244a	0.4874a
0.09a ⁴	L			0.3191a	0.4701a	0.3267a	0.4933a
	G=O			0.2455a	0.4957a	0.2349a	0.5313a
0.1a ⁴	L					0.3031a	0.5605a
	G=O					0.2444a	0.5861a

L : Löschian; G=O : Greenhut = Ohta

Table 2 . Areas in the equilibria for the three kinds of shapes

Fixed Costs	Model	Triangle	Square	Hexagon
0.01a ⁴	L	0.0482a ²	0.0477a ²	0.0475a ²
	G=O	0.1049a ²	0.1173a ²	0.1260a ²
0.02a ⁴	L	0.1061a ²	0.1043a ²	0.1037a ²
	G=O	0.1846a ²	0.2034a ²	0.2168a ²
0.03a ⁴	L	0.1734a ²	0.1694a ²	0.1679a ²
	G=O	0.2643a ²	0.2872a ²	0.3042a ²
0.04a ⁴	L	0.2515a ²	0.2439a ²	0.2410a ²
	G=O	0.3485a ²	0.3733a ²	0.3928a ²
0.05a ⁴	L	0.3430a ²	0.3295a ²	0.3245a ²
	G=O	0.4410a ²	0.4648a ²	0.4856a ²
0.06a ⁴	L	0.4521a ²	0.4290a ²	0.4208a ²
	G=O	0.5464a ²	0.5649a ²	0.5854a ²
0.07a ⁴	L		0.5472a ²	0.5337a ²
	G=O	0.6721a ²	0.6781a ²	0.6960a ²
0.08a ⁴	L		0.6925a ²	0.6700a ²
	G=O		0.8120a ²	0.8229a ²
0.09a ⁴	L		0.8838a ²	0.8431a ²
	G=O		0.9831a ²	0.9778a ²
0.1a ⁴	L			1.0885a ²
	G=O			1.1901a ²

L : Löschian, G=O : Greenhut=Ohta

Table 2 the following. In the Lösch model, the area of hexagonal market area becomes the smallest at any level of fixed costs. On the other hand, however, in the Greenhut = Ohta model, as fixed costs increase the shape of market areas that make its area smallest varies. That is, when the fixed costs are below $0.07a^4$, the area of triangular market area becomes the smallest. When they are in between $0.08a^4$ and $0.09a^4$, the shape of square makes the area the smallest. And, it is a hexagon that makes the area of market areas the smallest when the fixed costs are beyond $0.09a^4$.

Therefore, we can draw from Table 2 the following conclusion. If the shapes of market areas under spatial competitive equilibrium is to be selected, they can not only be of hexagons but of triangles and squares. Which of these three shapes will be materialized depends on the manner of spatial competition and the level of production costs: If spatial competition is realized in the manner prescribed by Greenhut = Ohta and the fixed costs are below $0.07a^4$, the market area becomes a triangle; if the fixed cost is $0.08a^4$, it becomes a square; in the case of the fixed costs larger than $0.08a^4$, the market area takes the shape of a hexagon.

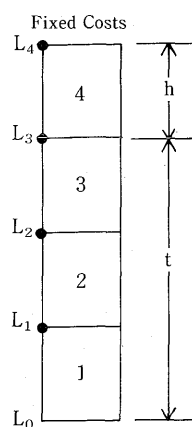
IV. THE CENTRAL PLACE SYSTEM THAT CONSISTS OF TRIANGULAR AND HEXAGONAL MARKET AREAS

From the considerations above, the cases in which triangular, square or hexagonal market areas appear in spatial competitive equilibrium were demonstrated. In this section, we will construct the central place system by combining both triangular and hexagonal market area network. Thus the conditions that will be discussed shortly are added, but, in principle, it is assumed that spatial competition is materialized in the manner of the Greenhut = Ohta model in which the triangular market area appears.

(1) Classification of Firms According to the Level of Fixed Costs

Now, it is assumed that all kinds of goods which are sold on a plane are produced with fixed costs alone and that

Fig. 1
Class-Determining in Goods and Levels of Fixed Costs



Notes : t: triangle, h: hexagon
The numbers 1, 2, 3 and 4
above refer to each hierarchy.

these fixed costs belong to one of the levels between L_0 and L_4 as shown in Figure 1.

Let us classify all goods into the four classes based on the levels of the fixed costs necessary for their productions. Let us call those goods whose fixed costs are just for the levels of L_1 , L_2 , L_3 , and L_4 the so-called class-determining goods. (Note that the levels of L_1 , L_2 , L_3 can be arbitrarily chosen) . Thus, let us suppose that those all firms that sell the goods whose fixed costs are in between L_0 and L_1 own the same location and the same shape and size of the market area as the firm selling the good which requires the fixed costs L_1 . Likewise, the firms selling goods at the L_1 - L_2 fixed costs own the same location and the market area as the firm selling the good which requires the L_2 fixed costs . For the levels of fixed costs of L_2 - L_3 and L_3 - L_4 , the similar relations are assumed to hold. Each firm selling the class-determining good behaves to seek the maximization of profits, but its profits are forced to zero due to spatial competition. On the other hand, those firms selling the goods other than class-determining goods can obtain to some extent the excess profits since they are given the market areas larger than the size of the zero-profit market area . In short, the size of the marketing area of each class-determining good is consistent with the lower limit of each marketing area.

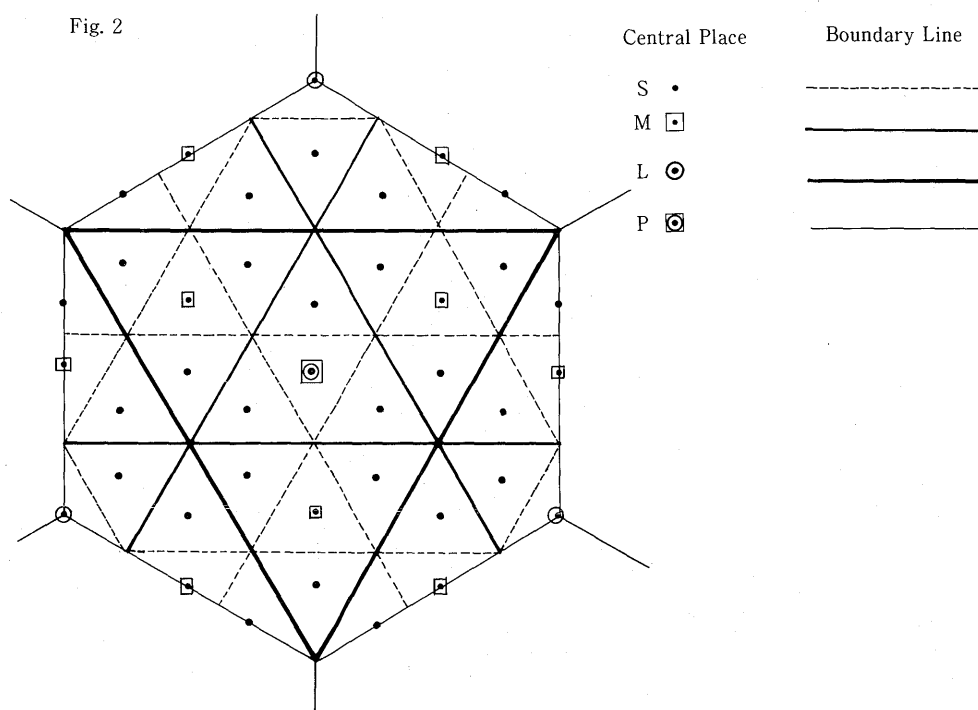
(2) Each Class of Market Area Networks and Its Transpositions

Now, the firms following the firm selling the goods with L_1 fixed costs form the first hierarchy and cover the whole plane as the market with the same location and the same market area. Thus they form the market area network of the so-called first hierarchy. The network of this market area turns out to be the finest . Next, those firms following the firms which sell the goods with the fixed costs L_2 consist of the second hierarchy, and cover the plane with the same location and the same market area, and make the market area network of the second hierarchy. Likewise, the market area networks of the third and the fourth hierarchies. The meshes of a net are made largest in the fourth hierarchical market area.

The central place system is thus constructed by transposing these four kinds of market area networks. Here, like Christaller's central place system, for the sake of simplicity, we will continue to transpose, starting from the first hierarchical market area network, in the order of higher priority, according to the following principle. That is, the firms belonging to the superior rank of hierarchy determine their locations consistent to the locations of all of the firms in the lower ranks. Therefore, the distribution of the locations and the size of the market area for the firms within the hierarchy superior to the second rank depend on the distribution of the locations of the firms belonging to the

first hierarchy and the lower ranks. Hence, there exist the possibilities for the size of the market areas of the firms belonging to the ranks superior to the second hierarchy to expand respectively. As a matter of course, this raises the possibility even for the firms selling class-determining goods to obtain excess profits.

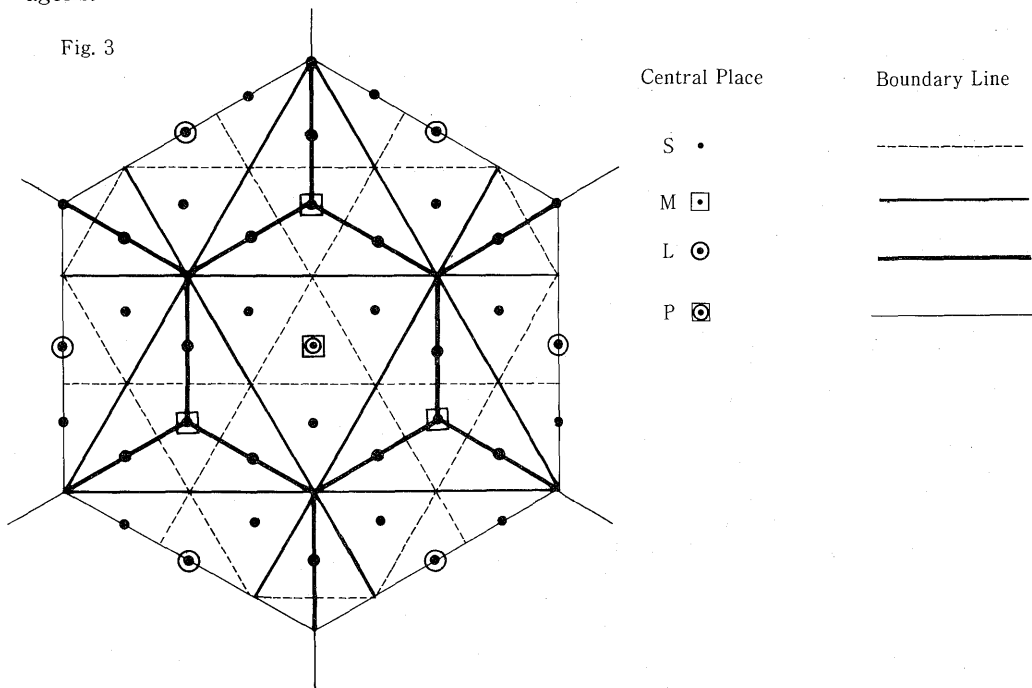
In light of the analysis in the third section, the firms selling the goods that require the $0-0.07a^4$ levels of the fixed costs, which cover 70 percent of the entire levels of fixed costs, own a triangular market area. Also, the firms selling those goods with the $0.09a^4-0.1a^4$ levels own a hexagonal market area. Corresponding to these facts, let us suppose that the market area for those firms with the first to the third hierarchies is a triangle, and that that for those firms belonging to the fourth hierarchy is a hexagon (See Fig. 2) . Figure 2 shows one of the central place system which was formed by transposing according to the aforementioned principle each hierarchy of these differently shaped market areas .



We will explain this system, using the symbols tK_2^t , K_3^t , and K_4^h , where K is the number of the market areas with the hierarchy of one rank lower. Here t and h relate to the shapes of triangles and hexagons. The levels of hierarchies are indicated by 2, 3 and 4. The left-sided t in tK_2^t implies that the shape of the first hierarchical market area is a

triangle. The central place system in Figure 2 is comprised of $tK_2^t=4$, $tK_3^t=4$, and $tK_4^t=2$. We indicate by $\odot$ the point at which the locations of the firms of four different hierarchies are meeting, and designate P as the central place. Hence the central place P is the largest central place that supplies all kinds of goods. In what follows, we indicate as the point where the locations of the three hierarchical firms meet, and make L the central place. $\odot$ is the point where the two hierarchical firms overlap, and M is the central place. The point at which only the first hierarchical firms locate is indicated by $\bullet$, and the central place is shown as S. The numbers of each central place are found as follows: The P central place is 1, the L central place is 1, the M central places are six, and the S central places are 24. Thus the following series appear: 1P-1L-6M-24S. Moreover, the number of the lower ranked market areas that the market area of the P central place contains are: the market areas of the L central place are 2, those of the M central place are 8, those of the S central place are 32. The next series thus appear: 1P-2L-8M-32S. These series would not appear when we try to construct the central place system only by hexagonal market areas.

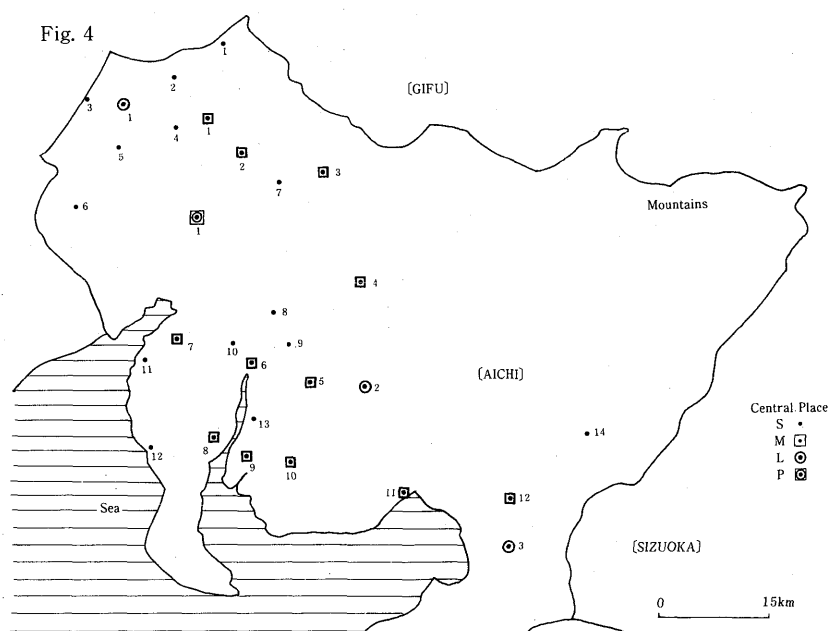
Figure 3 shows one of the central place system, constructed on the assumptions that the shape of the market area of the firms belonging to the bottom two hierarchies are triangles and that that of the firms belonging to the top two hierarchies are hexagons.



In terms of the symbols introduced earlier, it is constructed by $tK_2^t=4$, $K_3^h=2$, and $K_4^h=4$. The numbers of the central places are: the P central place is 1, the L central places are 3, the M central places are 3, and the S central places are 25. In this case, the series 1P-3L-3M-25S results. The numbers of the lower ranked market areas of each central place that the P central place contains are: those for the L central place are 4, those for the M central place are 8, and those for the S central place are 32, and the series 1P-4L-8M-32S was brought about. Again, these series would not appear in the case of the central place system constructed only by hexagons. Thus it is found that by introducing triangular market areas into the central place, the number of central places will be reduced in comparison with the central place system constructed only by hexagonal market areas.

V. An APPLICATION OF THE CENTRAL PLACE MODEL TO THE CITIES AND THE CITY-SIZE DISTRIBUTION IN AICHI PREFECTURE

In this section, we will attempt to apply the hitherto constructed central place system to the actual data. Here an application of our central place system to the cities and the city-size distribution in Aichi prefecture in Japan will be considered. The shape of Aichi prefecture is shown in Figure 4.



The northeast region of the prefecture is mountainous and its southern region is the sea. The majority of cities are distributed on the plane which stretches from the northwest to the southeast directions.

Let us measure the concept of centrality of each city in terms of the employees in the wholesale industry in each city. The number of employees in 1979 in the wholesale industry in each city is listed in Table 3¹⁰⁾.

Utilizing the double-log table and measuring the number of employees in the ordinate and the descending ranks in the abscissa, Figure 5 was drawn.

It was found from Figure 5 that the cities in Aichi prefecture consist of the four hierarchies. Using the same symbols as in the previous section, we designate $\bigcirc$ as the largest central place P, and the others as $\odot$, $\square$, and $\bullet$. Then the distribution of the central places of

Table 3 Number of Employees in Wholesale Industry in Aichi Prefecture, 1979

NAME	NO	NUMBER
NAGOYA	P1	205510
TOYOHASHI	L3	11637
ICHINOMIYA	L1	7124
OKAZAKI	L2	6583
TOYOTA	M4	2771
GAMAGOURI	M11	2683
KOMAKI	M1	2339
SETO	M3	2171
ANJIO	M5	1969
KARIYA	M6	1770
KASUGAI	M2	1747
HANDA	M8	1532
NISHIO	M10	1437
TOYOKAWA	M12	1307
TOUKAI	M7	1082
HEKINAN	M9	1066
TSUSHIMA	S6	738
TOKONAME	S12	714
BISAI	S3	568
INUYAMA	S1	548
KOUNAN	S2	534
CHIRIU	S9	494
OWARIASAHI	S7	327
SHINSHIRO	S14	271
TAKAHAMA	S13	202
IWAKURA	S4	193
TOYOAKE	S8	160
CHITA	S11	134
INAZAWA	S5	60
OUBU	S10	44

each hierarchy is shown as in Figure 4. The numbers of the central places are: the P central place is 1, the L central places are 3, the M central places are 12, and the S central places are 14. This brings about the series of the number of central places: 1P-3L-12M-14S.

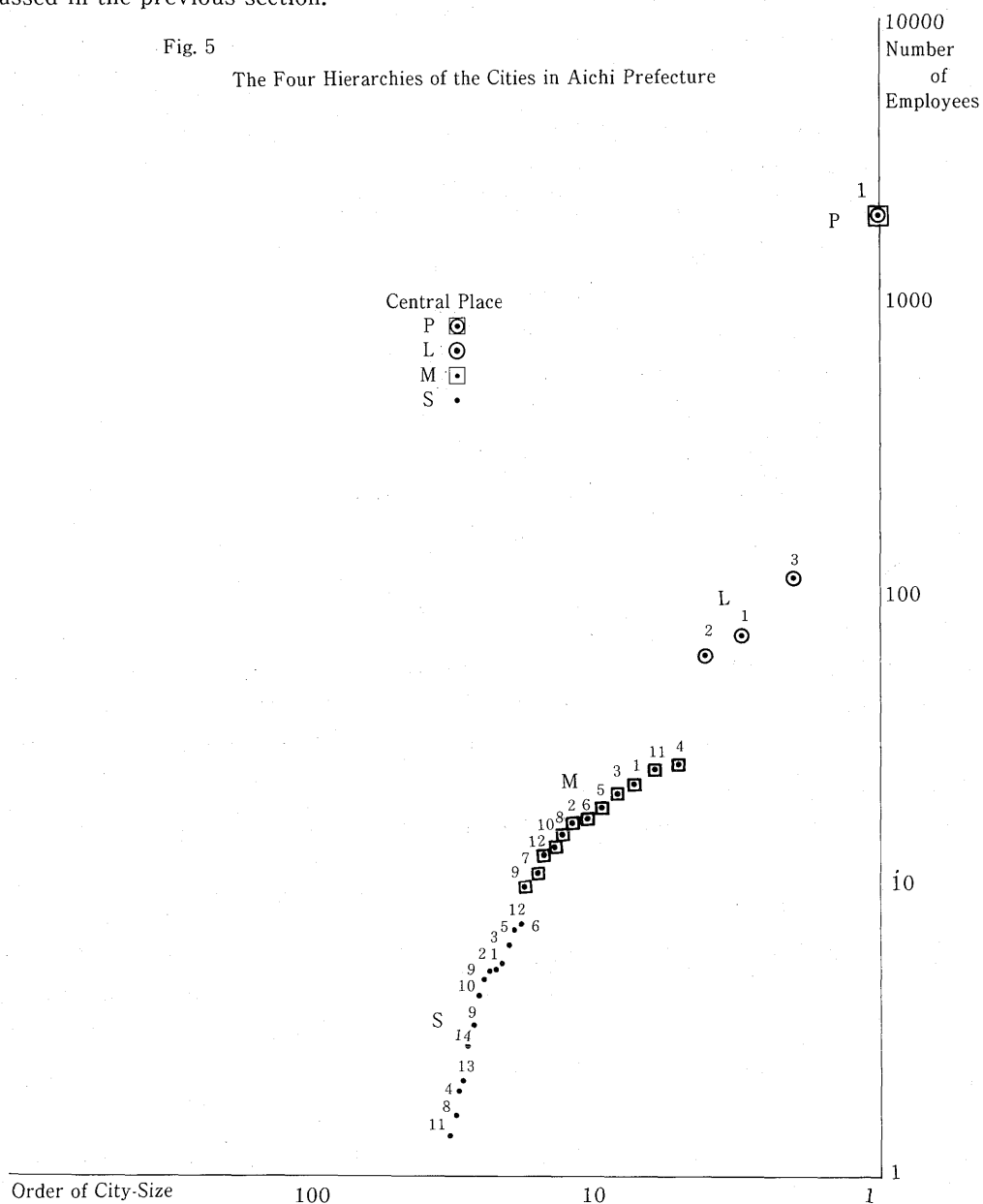
Let us construct the central place system that best explains the central places and their size distributions in Aichi prefecture. We will pay attention to the distances of the central places which belong to the same hierarchy and especially to the series of the number of the central places of each hierarchy, and will attempt to construct the central place system which generates most closely the actual situations.

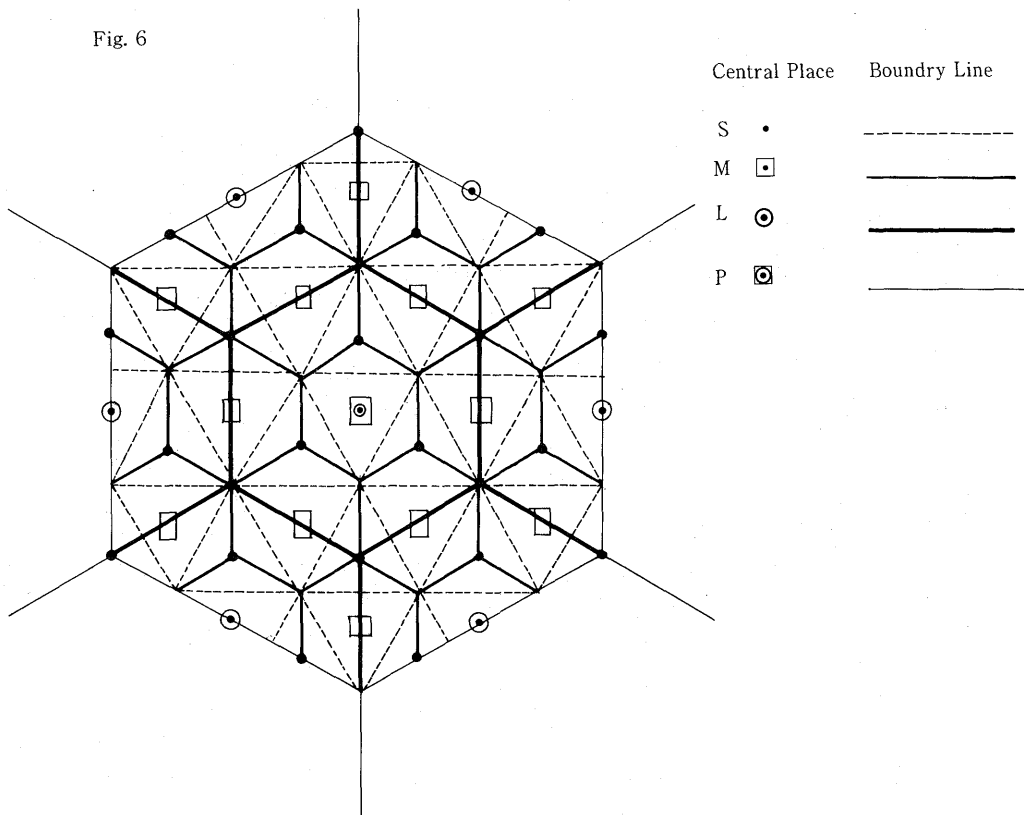
Now let us suppose the shapes and the sizes of the market areas of the firms within each hierarchy as follows. The shape of the market area of the first hierarchical firms is a triangle and the radius of its inscribed circle is 4.3 kilometers. The shapes of

the market areas for the second, the third and the fourth hierarchies are hexagons, and the radiuses of the inscribed circles are 7.5, 15, and 30 kilometers, respectively. Figure 6 shows the central place system that was constructed, through successively transposing the market area networks of the second, the third, and the fourth hierarchies, based on the first hierarchical market area network, according to the constructing principle discussed in the previous section.

Fig. 5

The Four Hierarchies of the Cities in Aichi Prefecture





In this central place system, the series of the number of the central places becomes 1P-3L-12M-16S. Since the series of the number of the central places for Aichi prefecture was 1P-3L-12M-14S, there exists the discrepancy of only two in the smallest number of the central places. Thus we can conclude that we obtained the almost consistent number of central places.

Next, let us look at the distances between each central place. Surveying the average distances between each size of central place in Aichi prefecture, it was found that the average distance among the L central places is 30 kilometers, that for the M central places is 14 kilometers, and that for the S central places is 11 kilometers. The average distances between each size of central place in the central place system in Figure 6 are the following: the average distance between the L central places is 30 kilometers, that for the M central places is 15 kilometers, and that for the S central places is 15 kilometers. We then conclude that despite a slight discrepancy (4 kilometers) between our model and the actuality of the average distance of the S central places we again obtained the almost consistent results.

VI. CONCLUDING REMARKS

From the considerations hitherto, we obtained the following two major conclusions. First, the shapes of the market areas in spatial competitive equilibrium are not necessarily confined to hexagons. Triangles and squares present another possibilities. Which shape will materialize depends on the manner of competition and the level of production costs. Secondly, the central place system constructed through using both triangles and hexagons has a more diversity than the central place system made of only hexagonal market areas. Hence, our model has a better and wider applicability for the analysis and the explanation of the actual cities and the city-size distributions.

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