

180° Bend Flow Meter I

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The purpose of the present experimental investigation is to reveal the performance of a 180° bend flow meter. Results of this study suggest that the most suitable measuring section of the differential pressure head between inside and outside walls of a bend would be such position as the bent-angle $\theta=45^\circ$ to 67.5° .

From these results, we might tentatively conclude that if discharge coefficients C_d of these sections are placed at $C_d=1.0$, the rate of flux in a passage can be measured with accuracy to within about 3 percent error.

1. Introduction

An exact measurement of the rate of discharge in a pipeline is an important subject in industries; therefore, various types of flow meters have been employed according the specific functions needed. Widely used measuring methods are the head type flow meters, which utilize the differential pressure head generated between two sections of upstream and downstream positions in a pipeline.

The present bend flow meter also belongs to one of the head type meters, i.e., this device utilizes the difference of pressure which arises between inside and outside walls of a curved passage by the action of centrifugal force.

Since the earliest experimental data of bend flow meters were reported in the literature by Jacobs and Sooy¹⁾, many researchers have been studying them for a long time both experimentally and theoretically. The prominent papers are as follows: (a) that of Lansford²⁾, Addison³⁾, Itō⁴⁾ and authors^{5~7)} for the 90° bend flow meters, (b) that of Cortelyou⁸⁾, Tsuji⁹⁾ and Itō¹⁰⁾ for the 360° bend flow meters. However, besides Teshima¹¹⁾ and Maeda¹²⁾, very few have studied the 180° bend flow meter, so far as the author is aware, and there still remains various unexplored domains.

This investigation, therefore, aims at obtaining more fundamental knowledge of 180° bend flow meters and to confirm the relationship between the discharge coefficients and various influential factors. First we provided three kinds of 180° bends with square sectional flow area and with radius ratios of bend were 4.00, 6.00 and 8.00, and tests were carried out under the range of Reynolds numbers R_e of $10^4 \leq R_e \leq 9 \times 10^4$. The present paper is concerned with the experimental results.

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2. Nomenclature

- a : half width of square sectional bend
 C_{df} : discharge coefficient, [Eqs. (1), (2)]
 C_d : discharge coefficient, [Eq. (3)]
 C_D : discharge coefficient, [Eq. (4)]
 C'_D : discharge coefficient, [Eqs. (7), (7')]
 C_p : pressure coefficient, [Eq. (8)]
 D : inner-perimeter of square sectional bend ($=2a$)
 D_s : inner-perimeter of square sectional tangent
 H : pressure head on wall
 h : differential pressure head
 L : axial centerline length of passage
 n : radius ratio of bend ($=R/a$)
 Q : volumetric discharge
 R : radius of curvature of bent-centerline
 Re : Reynolds number ($=V_mD/\nu$, ν : kinematic viscosity)
 Re_c : critical Reynolds number
 S : cross-sectional flow area of bend
 V_m : mean axial velocity of flow
 θ : bent-angle from inlet of bend
 ζ : friction factor of bend, [Eq. (9)]
 λ : friction factor of straight passage

3. Definition of discharge coefficients

The definition of the discharge coefficients of bend flow meters have been formulated in three ways as follows:

- 1) discharge coefficient C_{df} , under a free-vortex flow theory^{2, 3)}, for a circular pipe,

$$C_{df} = Q\sqrt{n} / \{ \sqrt{2gh} \cdot S \cdot (n^2 - 1)(n - \sqrt{n^2 - 1}) \} \quad (1)$$

for a square duct,

$$C_{df} = 4Q\sqrt{n} / [\sqrt{2gh} \cdot S \cdot (n^2 - 1) \cdot \ln\{(n+1)/(n-1)\}] \quad (2)$$

- 2) discharge coefficient C_d , under a forced-vortex flow theory²⁾,

$$C_d = 2Q / (\sqrt{2gh} \cdot S \cdot \sqrt{n}) \quad (3)$$

- 3) discharge coefficient C_D , similar to a head type meter as an orifice,

$$C_D = Q / (\sqrt{2gh} \cdot S) \quad (4)$$

where Q is the volumetric discharge, h the differential pressure head measured between inside and outside bend-walls, S the flow area of a bend, n the radius ratio of a bend ($=R/a$) and g the acceleration of gravity.

From these equations, each discharge coefficient is related as follows:

for a circular pipe,

$$C_D = C_d\sqrt{n}/2 = C_{df}(n^2 - 1)(n - \sqrt{n^2 - 1})/\sqrt{n} \quad (5)$$

for a square duct,

$$C_D = C_d \sqrt{n}/2 = C_{df}(n^2-1) \cdot \ln\{(n+1)/(n-1)\} / (4\sqrt{n}). \quad (6)$$

On the other hand, Lansford²³ introduced the following equation in his paper,

$$C'_D \sqrt{2D} = \sqrt{R} \quad (7)$$

where C'_D is the discharge coefficient, R the radius of curvature of a bend and D the inner diameter of a bend ($=2a$). Transforming the above equation we get

$$C'_D = \sqrt{R/(2D)} = \sqrt{R/(4a)} \quad (= \sqrt{n}/2). \quad (7')$$

Hence, it can be seen that when $C_d=1.0$ the Lansford's discharge coefficient C'_D must be equal to C_D .

4. Experiment

4.1 Experimental apparatus

The arrangement of experimental apparatus used for this study is illustrated in Fig.1. Water pumped up from the reservoir by a pump ① flows continuously through an overflow tank ② in which the water level is held constant by an overflow pipe. It then flows through a discharge control valve ③, a rectifying tank ④, an upstream tangent ⑤, a bend ⑥ and a downstream tangent ⑦ into a metering tank ⑧, and then, after weighing, it is returned to the reservoir. The water temperature was measured with a mercury thermometer ⑨ during each test.

The axial length of the upstream and downstream tangents, ⑤ and ⑦, were made about 61 and 77 times as long as their inner-perimeters D_s respectively in order to obtain a fully developed flow and to avoid the effects of bend. Additionally four piezometers (1.2 mm bore) were fitted equally on the perimeter of both measuring sections A and B, whose positions were placed about 47 and 61 times D_s from the inlet and outlet of the bend respectively, so that each mean wall-pressure head was measured. A vinyl chloride tubing pipe having 65.0 mm inner diameter was attached to the outlet of the downstream tangent ⑦.

4.2 Bend

Square cross-sectional 180° test bends were constructed from an acrylic resins board and their shape and geometric parameters

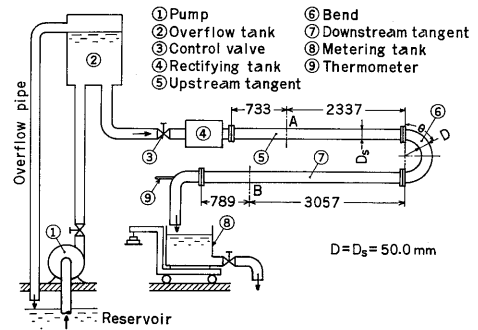


Fig.1 Experimental apparatus

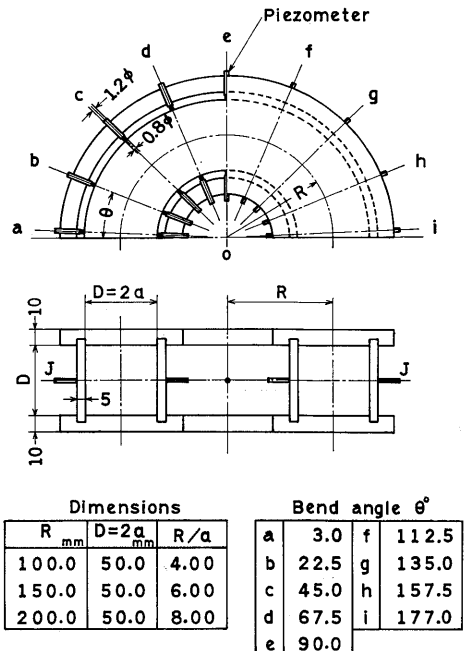


Fig.2 180°-bend

are shown in Fig. 2. As indicated in this figure, three different bend radius ratios R/a were used — 4.00, 6.00 and 8.00, and piezometers (0.8 mm bore) were fitted on the inside and outside walls of each measuring section as shown with the letters *ao* through *io*.

4.3 Measuring procedure

Wall-pressure heads H were measured with the above mentioned piezometers to an accuracy of the order of $1/100$ mm. The rates of discharge Q were measured by the weighing method within one percent error, and the Reynolds numbers R_e in all experimental runs were in the range of $10^4 \leq R_e \leq 9 \times 10^4$.

5. Experimental results and consideration

5.1 Discharge coefficient

5.1.1 Discharge coefficient vs. Reynolds number

In this paper, the discharge coefficients C_d and C_b defined by Eqs. (3) and (4) are mainly used.

Figure 3 shows the results about the discharge coefficients C_d of the test bend flow meters against the Reynolds numbers R_e for seven bent-angles θ and the three radius ratios of bend R/a .

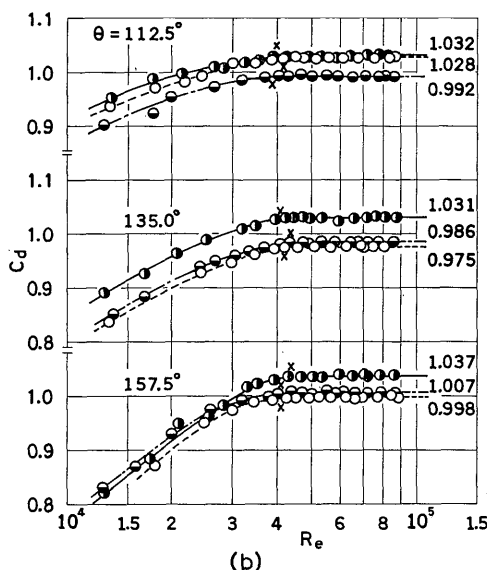
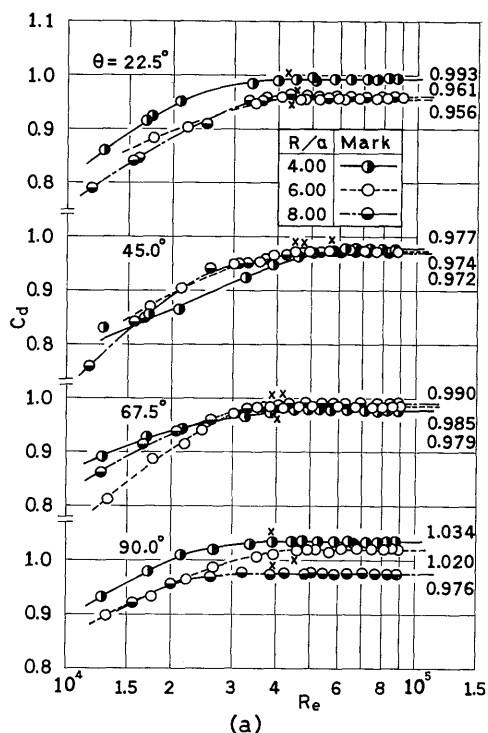


Fig. 3 Discharge coefficients C_d vs. Reynolds numbers R_e

From this figure, it can be seen that all of the discharge coefficients C_d increase with increasing Reynolds numbers R_e , and that they tend to converge to each constant value from the marked Reynolds number with an “x” on each curve. Hence, these marked Reynolds numbers are taken as the critical Reynolds numbers R_{ec} , and each numerical value along with curves is the average of all values of C_d for R_e

above each R_{ec} .

Examining this figure, there are some differences in the effects of bent-angles θ and the radius ratios R/a on the discharge coefficients C_d , but both C_d curves of $\theta=45^\circ$ and 67.5° sections seem to approach a single curve.

For further analysis, we summarize above mentioned values of C_d , R_{ec} in Table 1 together with that of C_D , C'_D and each percentage decrease $\Delta C_D/C'_D$, where $\Delta C_D = C_D - C'_D$. These values are also plotted in Fig. 4 as a function of the bent-angles θ and the radius ratios R/a . The two-dot-dash lines in this figure indicate the values of C'_D defined by Eqs. (7) and (7').

It can be seen from the figure of C_D , $\Delta C_D/C'_D$ vs. θ that all discharge coefficients C_D for $22.5^\circ \leq \theta \leq 67.5^\circ$ are smaller than C'_D and the percentage decreases $\Delta C_D/C'_D$ are almost all within -4 percent. Those for the positions of $\theta=45^\circ$ to 67.5° , $\Delta C_D/C'_D$ are within about -3 percent.

Thus we may be able to conclude that if the discharge coefficients C_d at the sections of $\theta=45^\circ$ to 67.5° are taken as $C_d=1.0$, the rate of discharge in a passage can be measured exactly within about 3 percent error above each critical Reynolds number R_{ec} .

However, it seems that the minimum critical Reynolds numbers occur in the range of $\theta=67.5^\circ$ to 112.5° , but the maximum ones at about $\theta=45^\circ$. As seen from these results, the most suitable set-position of piezometers would be around $\theta=67.5^\circ$ section of bend in terms of a performance of bend flow meter.

5.1.2 Discharge coefficient vs. radius ratio of bend

Figure 5 is a plot of the discharge coefficients C_D obtained from this investigation and that of other experimenters versus the parameters $\sqrt{R/4a}$ or radius ratios R/a , and a one-dot-dash line given in this figure indicates the Lansford's equation $C'_D = \sqrt{R/4a}$.

From this figure, these coefficients C_D

Table 1 Values of C_d , C_D , $\Delta C_D/C'_D$ and R_{ec} against R/a and θ

R/a	θ°	22.5	45.0	67.5	90.0	112.5	135.0	157.5	C'_D
4.00	C_d	0.993	0.977	0.979	1.034	1.032	1.031	1.037	1.000
	C_D	0.993	0.977	0.979	1.034	1.032	1.031	1.037	
	$\Delta C_D/C'_D(\%)$	-0.70	-2.30	-2.10	3.40	3.20	3.10	3.70	
	$R_{ec} \times 10^5$	4.25	5.80	4.00	3.90	4.00	4.10	4.60	
	$\Delta C_D = C_D - C'_D$								
6.00	C_d	0.956	0.974	0.985	1.020	1.028	0.975	0.998	1.225
	C_D	1.170	1.193	1.207	1.250	1.259	1.194	1.222	
	$\Delta C_D/C'_D(\%)$	-4.49	-2.61	-1.47	2.04	2.77	-2.53	-0.24	
	$R_{ec} \times 10^5$	4.35	4.50	3.90	4.50	4.15	4.25	4.15	
	$\Delta C_D = C_D - C'_D$								
8.00	C_d	0.961	0.972	0.990	0.976	0.992	0.986	1.007	1.414
	C_D	1.359	1.374	1.400	1.380	1.403	1.394	1.424	
	$\Delta C_D/C'_D(\%)$	-3.89	-2.83	-0.99	-2.40	-0.78	-1.41	0.71	
	$R_{ec} \times 10^5$	4.50	4.75	4.15	3.95	3.90	4.40	4.10	
	$\Delta C_D = C_D - C'_D$								

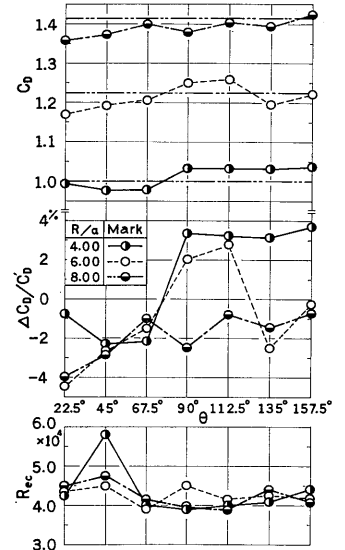


Fig. 4 Relations between C_D , $\Delta C_D/C'_D$, R_{ec} and θ

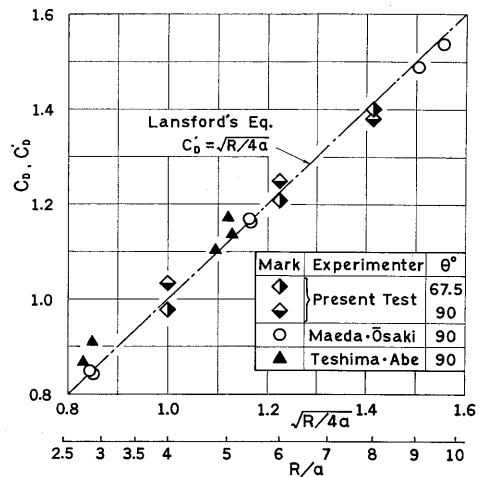


Fig. 5 Discharge coefficients C_D , C'_D vs. $\sqrt{R/4a}$, R/a

are close to the line C'_d except at two or three points, so it may be considered effective to some degree for the discharge coefficients C_d to be approximated by the Lansford's equation.

5.2 Pressure distribution on bend-walls

5.2.1 Definition of pressure coefficient

The pressure coefficient C_p is introduced as follows:

$$C_p = \frac{\lambda L_i}{2a} - \frac{(H_A - H_i)}{V_m^2 / (2g)}, \quad (8)$$

where λ is the friction factor for a straight passage, L_i the axial centerline length measured from the section A of the upstream tangent ⑤ to each pressure measurement section of test bends, H_A the mean wall-pressure head of section A, H_i the wall-pressure head at each pressure measurement section as seen in Fig.2, a the half width of square sectional bend, V_m the mean axial velocity of flow and g the acceleration of gravity.

5.2.2 Bend-wall's pressure distribution

The pressure coefficients C_p on bend-walls at each of the nine measurement sections were calculated for the two Reynolds numbers, $Re = 5.0 \times 10^4$ and 8.0×10^4 , and the results are shown in Figs. 6 (a) and 6 (b).

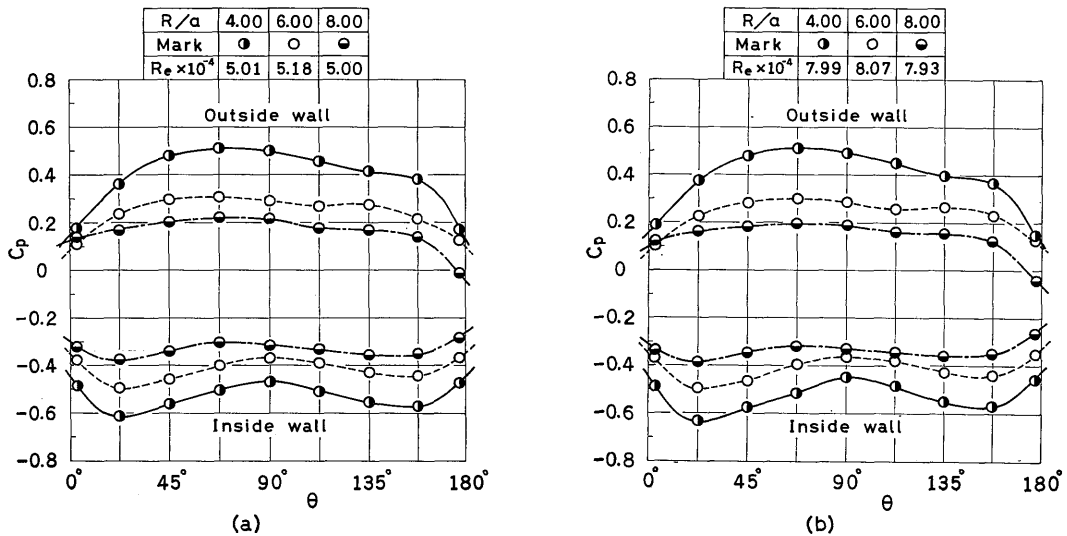


Fig. 6 Pressure distribution of bend-walls

Examining these figures there are no substantial differences between the profiles of C_p for the two conditions of low and high Reynolds numbers. As seen in these plots, the patterns of outside-wall C_p are seen as humped curves having the maximum values at the positions of $\theta = 67.5^\circ$, and moreover they are slightly constricted at the position $\theta = 135^\circ$ for $R/a = 4.00$, and at $\theta = 112.5^\circ$ for $R/a = 6.00, 8.00$. These phenomena cannot be seen in the literature as far as the author is aware; thus, we hope to find the causes of them in the future.

Similarly for the curves of inside-wall C_p , it can be seen that the values of C_p

are minimized at $\theta=22.5^\circ$ (for all R/a as a group), they then increase gradually with increasing θ and arrive the maximum at $\theta=67.5^\circ$ for $R/a=4.00$ and at $\theta=90^\circ$ for $R/a=6.00, 8.00$. After that they tend to decrease a little till $\theta=157.5^\circ$ and again they rise rapidly. These phenomena also cannot be found in the literature.

5.3 Friction factor of bend

A number of treatises^{13~15)} of the friction factors of bend have been published over a long period of time and many experimental equations have been proposed. In this paper the friction factor of bend ζ is defined as follows:

$$\zeta = \frac{H_A - H_B}{V_m^2 / (2g)} - \frac{\lambda(L_u + L_d)}{D_s} \quad (9)$$

where H_A and H_B are the mean wall-pressure heads of the sections A and B shown in Fig.1, V_m the mean axial velocity of flow, λ the friction factor for a straight passage, L_u the axial length measured from the section A to the inlet of bend, L_d the axial length between the outlet of bend and the section B, and D_s the inner-perimeter of a square sectional tangent.

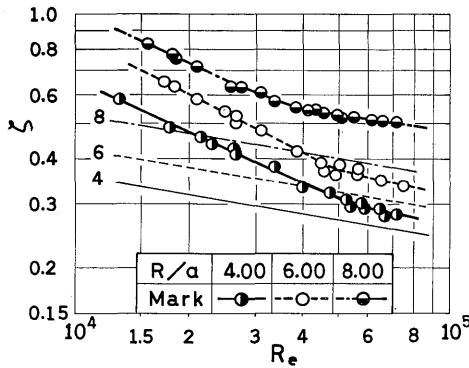


Fig. 7 Friction factor of bend ζ vs. Reynolds numbers Re

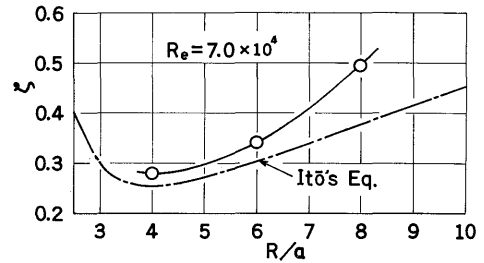


Fig. 8 Relations between ζ and R/a

The results of the friction factors of bend ζ are plotted against the Reynolds numbers Re in Fig.7 for three R/a . The lines "4, 6 and 8" in this figure indicate the following Itō's experimental equation¹⁵⁾ for each bend of $R/a=4.00, 6.00, 8.00$ respectively,

$$\zeta = 0.00241 \alpha \theta Re^{-0.17} (R/a)^{0.84} \quad ; \quad Re(a/R)^2 > 91 \quad (10)$$

where $\alpha = 1 + 116(R/a)^{-4.52}$ (for $\theta=180^\circ$).

From this figure, it can be seen that all ζ 's decrease considerably with increasing Re . The decrease is slow at Reynolds numbers $Re=(4.0, 5.0 \text{ and } 5.5) \times 10^4$ for $R/a=8.00, 6.00$ and 4.00 respectively, then the curves become approximately parallel to the above mentioned Itō's lines "8, 6 and 4".

Figure 8 shows the relationship between ζ and R/a for $Re=7.0 \times 10^4$. A one-dot-dash line in the figure indicates Eq. (10). As is obvious from Figs.7 and 8, there are great differences between our experimental results and Itō's calculated values, and all ζ 's obtained for $Re=7.0 \times 10^4$ are larger than that of Itō's equation (about 10,

12 and 31 percent for $R/a=4.00$, 6.00 and 8.00 respectively). The cause of these deviations is not clear, but we can estimate that it depends more or less on the second type secondary flow¹⁶⁾ appearing in the flows through non-circular cross-sectional pipes.

6. Conclusion

The experimental investigations on the 180° bend flow meters, whose dimensions are $R/a=4.00$, 6.00 , 8.00 and $10^4 \leq R_e \leq 9 \times 10^4$, lead to the following conclusions:

- 1) The discharge coefficients C_d at the measuring sections of $\theta=45^\circ$ and 67.5° can be seen to approximate a single curve regardless of R/a and if all C_d 's of above sections are taken as $C_d=1.0$, we can measure the rate of discharge to within about 3 percent error.
- 2) Nearly all the minimum critical Reynolds numbers R_{ec} of the test bends arise in the range of $67.5^\circ \leq \theta \leq 112.5^\circ$, but the maximum ones appear at $\theta=45^\circ$.
- 3) For the pressure distribution on bend-walls, we found such interesting phenomena that the patterns of C_p 's on outside wall are slightly constricted in the positions of $\theta=112.5^\circ$ to 135° and that of C_p 's on inside wall form upward convex curves having the top at $\theta=67.5^\circ$ to 90° . These results cannot be seen in the literature as far as the author is aware.
- 4) The friction factors of bend ζ differed from that of Itô's Eq. (10). The cause may be attributed more or less to the second type secondary flow.

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