

Merchant-Rankine Approach to Plate Girders

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This report is intended to extend the Merchant-Rankine formula for evaluation of the ultimate strength of plate girders. The reviewed experimental values which are compared with this extended formula (M.R type formula) are 91 test girders in bending and 51 in shear, respectively. The system factors in the M.R type formula are given by the statistical analysis, which well explained the mean and lower boundary of the experimental values, and this formula also compared with the others.

1. Introduction

When the experimental values of the ultimate strength of the steel structural member are expressed in the orthogonal coordinate system with the buckling parameter as the abscissa and the ultimate strength as the ordinate, they closely approximate the elastic buckling curve or the postbuckling curve in the elastic range. When the buckling parameter is closer to zero, they belong to the plastic strength or the yield strength. In the inelastic range, they obey a certain transition curve. For this curve, for example, the well known formulas of Rankine-Goldon, Tetmajer and Johnson have been used many times in the past to suggest the ultimate strength of columns.

The same familiar types of Rankine-Goldon formulas have been employed to estimate the instability of beams. These curves are expressed as simple equations, and the yield strength point and the elastic buckling curve can be smoothly conjoined. Moreover, it is considered that these formulas can be applied for the ultimate strength of the others. The Merchant-Rankine (M.R) formula which was proposed in the study on the instability of simple frames is a kind of Rankine-Goldon formula¹⁾.

The purpose of this paper is to propose the M.R type formula for the ultimate strength of plate girders.

2. M.R Type Formula

The Merchant-Rankine formula is indicated as following,

$$\frac{1}{\lambda_f} = \frac{1}{\lambda_p} + \frac{1}{\lambda_c} \quad \dots\dots\dots(1)$$

Indeed, this formula may be written

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$$\frac{\lambda_f}{\lambda_p} = \frac{1}{1 + \lambda_p/\lambda_e} \quad \dots\dots\dots(2)$$

where λ_f =failure load, λ_p =plastic load, and λ_e =elastic buckling load.

Since C. Massey proposed to introduce the idea of the system factor²⁾, Eq. (2) was recently extended to Eq. (3)

$$\frac{\lambda_f}{\lambda_p} = \frac{\beta}{[1 + \alpha(\lambda_p/\lambda_e)^p]^{\frac{1}{p}}} \quad \dots\dots\dots(3)$$

where p =system factor, and α, β =corrective coefficient.

Above demonstration is due to Klöppel-Unger³⁾ and Djalaly⁴⁾. The proposed ECCS formula⁵⁾ is also similar to Eq. (3).

In the field of column study, D. Allen extended this formula to the case of interaction between overall and local buckling of columns⁶⁾. The ECCS Multiple Column Curves will be expressed as M.R type formula per DIN 4114⁷⁾.

The authors applied the M.R type formula for the ultimate strength of the stiffened and unstiffened plates⁸⁾.

3. M.R Type Formula Approach to Plate Girders

—Plate Girders under Bending—

The collapse of plate girders under bending may be classified according to the following three buckling types of the compression flange. (1) Lateral buckling, (2) torsional buckling, and (3) vertical buckling. Equation (4) shows the M.R type formula as applied to plate girders under bending.

$$\frac{M_u}{M_y} = \left[\frac{1}{1 + (\sqrt{\alpha_M} \lambda)^{2p}} \right]^{\frac{1}{p}} \quad \dots\dots\dots(a)$$

$$\text{or} \quad \left(\frac{M_u}{M_y} \right)^p + \left(\alpha_M \frac{M_u}{M_e} \right)^p = 1 \quad \dots\dots\dots(b)$$

$$\text{where} \quad \lambda = \sqrt{M_y/M_e} = \frac{2\sqrt{3}}{\pi} \left(\frac{l_b}{b_f} \right) \sqrt{\sigma_{yf}/E} \quad \dots\dots\dots(c)$$

$$\left. \begin{aligned} \alpha_M &= R_M = \frac{h}{t_w} \sqrt{\sigma_{yw}/E \{12(1-\nu^2)/k\pi^2\}}, & R_M > 1 \\ \alpha_M &= 1 & R_M \leq 1 \end{aligned} \right\} \quad \dots\dots\dots(d) \quad (4)$$

in which M_u =ultimate bending moment, M_y =yield bending moment, M_e =elastic buckling moment (lateral buckling moment), l_b/b_f =ratio of the distance between the fixed points of plate girders, σ_{yf}, σ_{yw} =yield stress of the flange and web plates, respectively, h/t_w =width-thickness ratio of the web panel, and k =buckling coefficient of the web panel under pure bending with simply supported boundary condition.

Equation (4-a) means that λ , expressed by Eq. (4-c), controls the ultimate strength of plate girders under bending, and R_M , expressed by Eq. (4-d), is a corrective coefficient for λ . In other words, R_M affects the ultimate strength only in the case where $R_M > 1$ as the local buckling of plate girders.

It is here considered that the web buckling does not occur in plate girders with a horizontal stiffener.

The number of reviewed test plate girders⁹⁾⁻¹¹⁾ swelled to 91 in which 31 plate girders with a horizontal stiffener were included, but the data were omitted in which torsional buckling of the compression flange plate may occur because $R > 0.7$.

Figures 1 and 2 show the plotted test results for plate girders under bending. Figure 2 differs from Fig. 1 only as to the parameter of the abscissa. The scatter

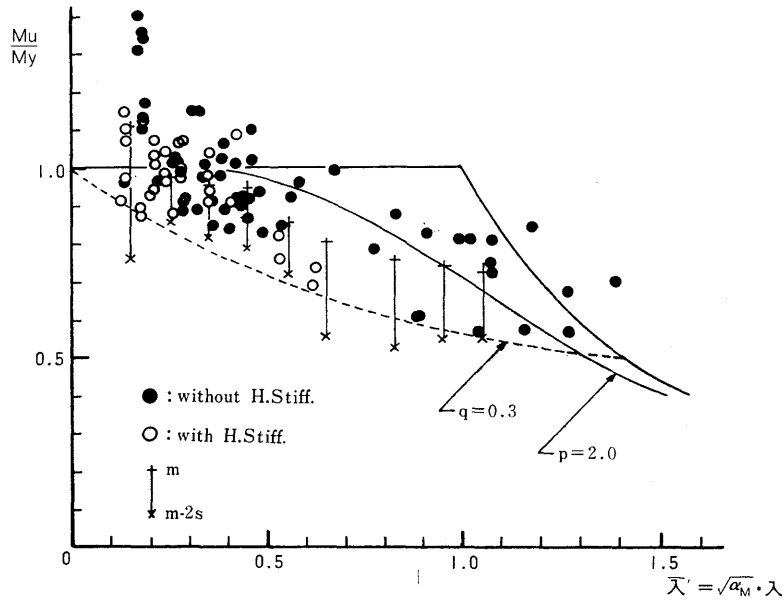


Fig. 1 Comparison of Proposed Formula with Reviewed Experimental Values (in Bending)

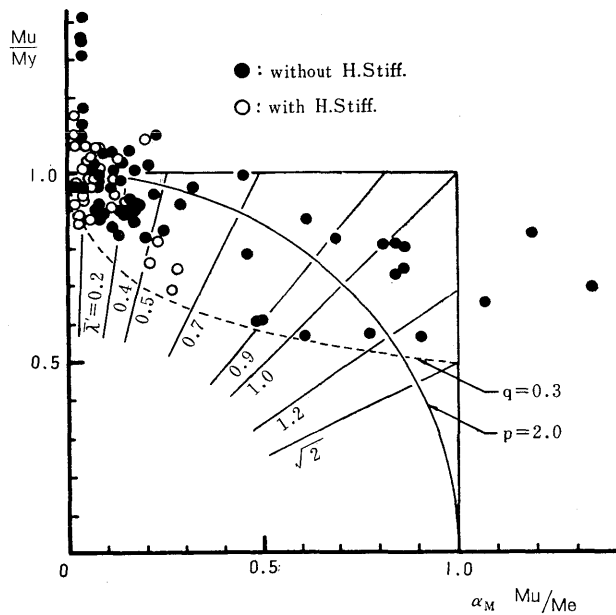


Fig. 2 Correlation of M_u/M_y and $\alpha_M(M_u/M_e)$ (in Bending)

of the test points was analyzed from a statistical point of view, giving the experimental means and means minus twice the standard deviations and the corresponding coefficient of variation. The test results are also compared with the proposed design formula of the M.R type in Eq. (4-a).

The single value $p=2.0$ was chosen in Eqs. (4-a) and (b) to ensure the resulting design curve corresponds to the mean value by the regression analysis.

The same familiar formula of Eq. (4-b) is proposed as Eq. (5), which takes into account the effective range of $\bar{\lambda}'$ ($\bar{\lambda}' = \sqrt{\alpha_M} \lambda$).

$$\frac{M_u}{M_y} + 0.5 \left(\alpha_M \frac{M_u}{M_e} \right)^q = 1 \quad \dots\dots\dots(5)$$

Although the constant value $q=2.0$ was chosen in Eq. (5) to explain the mean values, it is not suitable for comparison with $p=2.0$ in Eqs. (4-a) and (b) as shown in Table 1. The value $q=0.3$ may be chosen a suitable value to well explain the lower boundary.

Table 1 Comparison of Proposed Formula with Others
from Statistical Analysis (in Bending)

Proposed Formula	Number of Test Girders	System Factor	Mean (ex/th)	Standard Deviation	Coefficient of Variation
Eq. (4-a)	91	2.0	1.011	0.145	0.143
Eq. (5)	91	2.0	1.413	0.495	0.350
Cooper	89	—	1.017	—	0.130
Herzog	84	—	1.080	—	0.117
Owen-Rockey	67	—	1.007	—	0.122

The results of analysis from the statistical viewpoint of other proposed formulas¹²⁾ such as those of Cooper, Herzog and Owen-Rockey are also shown in Table 1. The accuracy of the M.R type formula does not compare unfavorably with other proposed formulas.

—Plate Girders under Shear—

The proposed M.R type formula is shown below as applied to plate girders under shear.

$$\frac{V_u}{V_p} = \left[\frac{1}{1 + (\sqrt{\alpha_v} R_v)^{2p}} \right]^{\frac{1}{p}} \quad \dots\dots\dots(a)$$

$$\text{or} \quad \left(\frac{V_u}{V_p} \right)^p + \left(\alpha_v \frac{V_u}{V_e} \right)^p = 1 \quad \dots\dots\dots(b)$$

$$\text{where} \quad R_v = \sqrt{V_p/V_e} = \frac{h}{t_w} \sqrt{\sigma_{yw}/(\sqrt{3} E) \{12(1-\nu^2)/\pi^2 k\}} \quad \dots\dots\dots(c)$$

$$\frac{1}{\alpha_v} = \gamma = \sqrt{3} \{1 + 4A_{cf}^2/(A \cdot A_w)\} \quad \dots\dots\dots(d)$$

in which A_{cf} , A_w =cross sectional area of the flange and web, respectively, A =cross sectional area of plate girders, k =buckling coefficient of the fixed panel under shear,

and the following approximate expression is used¹⁸⁾.

$$\begin{aligned} k &= 8.98 + 5.6/\alpha_s^2 & \alpha_s &\geq 1 \\ k &= 5.6 + 8.98/\alpha_s^2 & \alpha_s &\leq 1 \end{aligned} \quad \text{.....(e) \quad (6)}$$

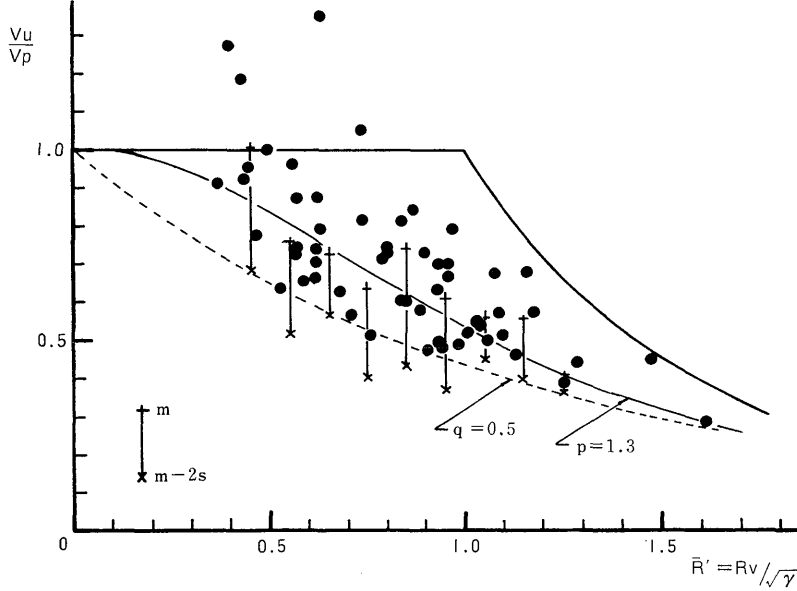


Fig. 3 Comparison of Proposed Formula with Reviewed Experimental Values (in Shear)

where α_s =aspect ratio of the web panel.

The corrective coefficient α_v in Eq. (6-d) means to take into account the area ratio of the compression flange to the web.

The 57 test results¹⁰⁾ which Hasegawa et al. obtained from the papers of Basler, Cooper, etc. are shown in Fig. 3. The correlation of V_u/V_p and $\alpha_v \cdot V_u/V_e$ is shown in Fig. 4. The system factor $p=1.3$ in Eqs. (6-a) and (b) explains the mean value as shown in Fig. 3 or 4.

The following expression is proposed to the lower boundary as shown by a dotted line in Figs. 3 and 4.

$$\frac{V_u}{V_p} + 0.85 \left(\alpha_v \frac{V_u}{V_e} \right)^{0.5} = 1 \quad \text{.....(7)}$$

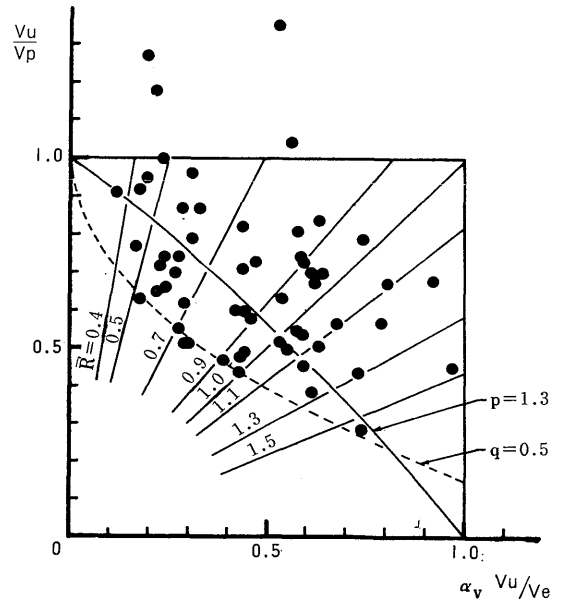


Fig. 4 Correlation of V_u/V_p and $\alpha_v V_u/V_e$ (in Shear)

Table 2 shows the comparison with the M.R type formula and the other ones^{1,2)} which were proposed by Basler and Rockey-Skaloud. The mean value is explained very well by the M.R type formula rather than the other ones.

Table 2 Comparison of Proposed Formula with Others from Statistical Analysis (in Shear)

Proposed Formula	Number of Test Girders	System Factor	Mean (ex/th)	Standard Deviation	Coefficient of Variation
Eq. (6-a)	57	1.3	1.000	0.200	0.200
Basler	57	—	1.020	0.268	0.262
Rockey-Skaloud	57	—	1.130	0.441	0.390

4. Conclusion

The M.R type formula discussed above is rearranged into the following expression for the estimation of the ultimate strength of the steel structural member.

$$\left(\frac{\lambda_f}{\lambda_p}\right)^p + \beta \left(\alpha \frac{\lambda_f}{\lambda_c}\right)^q = 1 \quad \dots\dots\dots(8)$$

in which p, q and α, β are the system factors and the corrective coefficients, respectively. They are summarized for the case of plate girders in Table 3.

Table 3 Summarized Coefficient for Plate Girders in Eq (8)

Plate Girders		p	q	β	α	$\sqrt{\lambda_p/\lambda_c}$
under Bending	Mean	2.0	2.0	1.0	α_M ; Eq. (4-d)	$\sqrt{M_y/M_e}$; Eq. (4-c)
	Lower Bound	1.0	0.3	0.5	α_M ; Eq. (4-d)	$\sqrt{M_y/M_e}$; Eq. (4-c)
under Shear	Mean	1.3	1.3	1.0	α_v ; Eq. (6-d)	$\sqrt{V_p/V_e}$; Eq. (6-c)
	Lower Bound	1.0	0.5	0.85	α_v ; Eq. (6-d)	$\sqrt{V_p/V_e}$; Eq. (6-c)

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