

The Presence of Anomalous Dimensions and a Logarithmic Cutoff

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It is shown that Bietti and Parisi's parametrization of an anomalous dimension parameter $\alpha_{2,v} \approx -0.035$ is essentially equivalent to an extremely large logarithmic cutoff, $\Lambda/\Lambda_0 \simeq e^{13}$, estimated in our previous work on the deep-inelastic contribution to the n - p mass difference.

Bietti and Parisi¹⁾ have shown that even a very small violation of scaling is enough to give a correct value of the n - p mass difference by assuming an anomalous dimension parameter $\alpha_{2,v} \approx -0.035$. In this report we note that their parametrization of $\alpha_{2,v}$ is essentially equivalent to a logarithmic cutoff $\ln(\Lambda/\Lambda_0) \simeq 13$, which was discussed in our previous work,²⁾ where Λ and Λ_0 stand for the upper and lower limits of the momentum variable. In other words, the presence of anomalous dimensions could give a plausible reason for an extremely large value of the logarithmic cutoff momentum.

Of course, we here assume that the condition under which we can reproduce the correct sign of the observed n - p mass difference is satisfied. In fact, Bietti and Parisi¹⁾ have assumed such a condition in a specially simple case concerning the longitudinal photon cross section. In Ref. 2 the condition was also given by the somewhat general expression.

According to Bietti and Parisi's argument, the usual logarithmically divergent integral

$$I = \int_{\neq 0}^{\infty} \frac{1}{q^2} dq^2 \quad (1)$$

can be replaced by the integral

$$I' = \int_{\neq 0}^{\infty} \frac{1}{q^2} \left(\frac{q^2}{M^2} \right)^{-\delta} dq^2 \quad (\delta = -\alpha_{2,v} > 0). \quad (2)$$

when there exists an anomalous dimension. Let the upper and lower limits of the integration variable q^2 be Λ^2 and Λ_0^2 ; it follows that

$$I \simeq \ln(\Lambda^2/\Lambda_0^2) \quad (3)$$

and

$$I' \simeq -\frac{1}{\delta} (\Lambda_0^2/M^2)^{-\delta} - (\Lambda^2/M^2)^{-\delta}. \quad (4)$$

When obtaining Eq. (3), whether we use a straight cutoff, a Feynman cutoff, or any power of Feynman's factor³⁾ makes little difference in case of $\Lambda^2 \gg \Lambda_0^2$;

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$$\begin{aligned} \int_{\Lambda_0^2}^{\Lambda^2} \frac{1}{q^2} dq^2 &\approx \int_{\Lambda_0^2}^{\Lambda^2} \frac{1}{q^2} \left(\frac{\Lambda^2}{q^2 + \Lambda^2} \right) dq^2 \\ &\approx \int_{\Lambda_0^2}^{\Lambda^2} \frac{1}{q^2} \left(\frac{\Lambda^2}{q^2 + \Lambda^2} \right)^2 dq^2 \approx \dots \approx \ln(\Lambda^2/\Lambda_0^2). \end{aligned}$$

Similar relations hold with Eq. (4).

Assuming $\Lambda^2 \gg \Lambda_0^2 \approx M^2$, and considering that

$$\lim_{\delta \rightarrow 0} \frac{1}{\delta} (1 - x^{-\delta}) = \ln x, \quad (5)$$

we then obtain the relation

$$\delta \simeq 1/\ln(\Lambda^2/\Lambda_0^2) \quad (6)$$

for sufficiently small $\delta \ll 1$. Thus Bietti and Parisi's value of $\alpha_{2,v} (= -0.035)$ corresponds to $\ln(\Lambda/\Lambda_0) \simeq 1/2\delta \simeq 14$.⁴⁾

Therefore, if the violation of scaling is given by a very small negative value of the anomalous dimension parameter, we may say that the presence of such an anomalous dimension suggests a reason why we need an extremely large cutoff momentum in the logarithmically divergent integral.

Eq. (6) indicates that δ becomes smaller with increasing cutoff momentum. If we apply the above reasoning to the case of the electron self-energy, the conventional formula,⁵⁾ $\ln(\Lambda/m_e) \simeq 137$, leads to $\delta \simeq 0.0036$. That is to say, if there is a violation of scaling in this case also, it would be about one-tenth of that of the nucleon.

References

- 1) A. Bietti and G. Parisi, Phys. Letters **43B** (1973), 427.
- 2) H. Katsumori, Phys. Rev. **D6** (1972), 1438.
- 3) The factor $[\Lambda^2/(q^2 + \Lambda^2)]^2$ is introduced as a finite size effect of the constituents. See M. S. Chanowitz and S. D. Drell, Phys. Rev. Letters **30** (1973), 807 for further discussion. See also K. Matumoto, Prog. Theor. Phys. **47** (1972), 1795.
- 4) A slight numerical discrepancy between this figure and our estimate in Ref. 2 comes from how we choose a visual fit curve through the experimental points of the difference of structure functions $\nu W_{2,p} - \nu W_{2,n}$.
- 5) See, for example, S. S. Schweber, An Introduction to Relativistic Quantum Field Theory (Harper and Row, New York, 1961), p. 541.