

Thermal Convection within the Earth's Mantle

Jun IRIYAMA*

(Received September 1, 1971)

A linear stability problem for the mantle of the earth is considered. Upper and lower mantle convection currents are assumed to reflect a rapid change in the viscosity at transition zone, i.e., Bullen's region C. The theory of mantle convection proposed by Tozer is adopted as a basis for the present investigation.

The temperature at the bottom of the upper mantle convection cell is about 1450–1650°K regardless of conduction transfer and the thickness of the convecting zone. For appropriate values of physical parameters it is found that the upper mantle of the earth is considerably less thermally stable than the lower mantle.

1. Theoretical Background

The onset of convection at some zone within a chemically homogeneous planet is governed by the steepness of the temperature gradient dT/dr relative to the adiabatic gradient $(dT/dr)_{ad}$ at that zone. If dT/dr were less than or equal to $(dT/dr)_{ad}$ under these conditions, then there would be no energy transfer by convection. Convection current carries large amounts of energy from the interior of the earth, and therefore a steady situation will be realized eventually, in which both conduction and convection compete as heat transport mechanism (Cox and Giuli¹⁾). The actual $T(r)$ curve within the convective zone has a slope intermediate between that of the conductive $T(r)$ curve and that of the adiabatic $T(r)$ curve.

We assume that the thermal convection will occur if the Rayleigh number

$$Ra = g\alpha d^3 \delta T / \kappa \nu \quad (1)$$

exceeds a critical value of 2300, where g is the acceleration of gravity, α the thermal coefficient of expansion ($= 2 \times 10^{-5}/deg$), d the thickness of the layer, δT the temperature difference between the upper and lower boundary of the considering layer calculated from conduction theory, κ the thermal diffusivity, and ν the kinematic viscosity.

The theory of mantle convection proposed by Tozer²⁾ is adopted here as a basis for the present calculation. We assume that convection is to reduce the temperature at the bottom of the convecting layer a minimum. If heat is fully transported to the upper part of the layer by convection current, then the temperature at the lower boundary of the convecting layer will become a minimum. The ratio of the heat brought to the upper surface of a convecting system to the heat that is brought in conduction is given in terms of the Nusselt number, a dimensionless quantity defined as (e.g., Knopoff³⁾)

$$Nu = H / (\kappa \rho C_p \delta T / d) \quad (2)$$

where H is the total heat escaping at the upper surface, C_p the specific heat at constant

*Department of Fundamental Science

pressure, and ρ the density. Thus, the temperature at the bottom of convection cell will decrease in proportion to the Nusselt number in the system and is (Tozer²⁾)

$$T_m = T_{UB} + \delta T / Nu \quad (3)$$

where T_{UB} is the temperature at the upper boundary of the fluid layer calculated from conduction theory. The Nusselt number is calculated from the Rayleigh number in the convecting system by using the empirical results by Silveston.⁴⁾

2. Model

We consider the mantle convection model under the fixed boundary conditions and internal heat generation. The upper and lower mantle convection currents are assumed to reflect a rapid change in the viscosity at transition zone, i.e., Bullen's region C. Two models are considered with respect to the conductive temperature distribution within the mantle. These are classified into the following types: (1) high temperature type (model H1 ; Iriyama,⁵⁾ his model RC-U2), and (2) low temperature type (model L1). These are illustrated in Figures 1 and 2.

The thermal conductivity of the medium is given by

$$K_T = C + 16n^2ST^3/3\varepsilon \quad (4)$$

where C is the ordinary lattice conductivity ($=2.5 \times 10^5$ ergs/cm/sec/deg), n the refractive index ($=1.7$), S the Stefan-Boltzmann constant, and ε the opacity ($=10/\text{cm}$).

We assume that the shear viscosity of the medium is (Tozer²⁾)

$$\eta = 700T \exp(E/kT) \quad (5)$$

where E is the activation energy, and k the Boltzmann's constant. It is supposed that a rapid change of the viscosity occurs in the transition zone of the mantle. Thus, in the calculations the activation energy within upper mantle convection zone is set equal to 4.0 eV. At the upper boundary of lower mantle convection zone it is set equal to 5.6 eV. The lower (upper) boundary of the upper (lower) mantle convection cell is assumed to be at depths between 500 and 700 km. The lower boundary of the lower mantle convection is assumed to be at the core-mantle boundary.

3. Results and Conclusions

The parameters (UB , d , T_{UB} , and T_{LB}) adopted in the calculations and the physical values (η , Ra , Nu , and T_m) obtained from the calculations are illustrated in Tables 1 and 2, and Figures 1 and 2. The blank space in the table shows that convection current is absent from the model. In the calculation of Rayleigh number the viscosity is set equal to its value at the upper boundary of the convecting region.

The conclusions which can be made are the following:

1. The temperature at the bottom of upper mantle convection cell is about 1450–1650°K regardless of the conduction equation solution and the thickness of the convecting layer.

2. If the temperature at the bottom of upper mantle convection cell is smaller than about 1450°K, then the lower mantle convection does not occur.

3. Variation of depth of the lower boundary of the upper mantle convection between 500 and 700 km affects little on the temperature within the convecting region of the

Table 1 Data for hot model (model H1)

Model	$UB,^*$ km	$d,$ km	$T_{UP},$ °K	$T_{LB},^{**}$ °K	$\eta,$ poise	Ra	Nu	$T_m,$ °K
H1-500'	40 500	460 2400	1050 3150	3150 4070	1.18×10^{25}	1.00×10^2		
H1-500	60 500	440 2400	1400 3150	3150 4070	2.48×10^{20} 1.68×10^{24}	2.23×10^6 3.48×10^4	11.52 4.51	1552 3354
H1-500''	70 500	430 2400	1610 3150	3150 4070	8.12×10^{18} 7.67×10^{22}	4.33×10^7 6.37×10^5	22.45 8.69	1679 3256
H1-600'	40 600	560 2300	1050 3260	3260 4070	1.18×10^{25} 3.87×10^{28}	1.90×10^4 1.21×10^5	3.94 5.99	1610 3395
H1-600	60 600	540 2300	1390 3260	3260 4070	3.13×10^{20} 2.58×10^{24}	3.55×10^6 1.88×10^4	12.79 3.93	1536 3466
H1-600''	80 600	520 2300	1790 3260	3260 4070	2.31×10^{17} 4.31×10^{21}	1.98×10^9 8.18×10^9	53.07 15.43	1818 3312
H1-700'	40 700	660 2200	1050 3345	3345 4070	1.18×10^{25}	3.23×10^2		
H1-700	60 700	640 2200	1400 3345	3345 4070	2.48×10^{20} 3.20×10^{24}	7.65×10^6 1.42×10^4	15.20 3.67	1528 3542
H1-700''	80 700	620 2200	1790 3345	3345 4070	2.31×10^{17} 4.56×10^{21}	3.71×10^9 6.67×10^9	61.11 14.74	1815 3394

* UB ; the depth of the upper boundary of fluid layer.

** T_{LB} ; the temperature at the lower boundary of the fluid layer calculated from conduction theory.

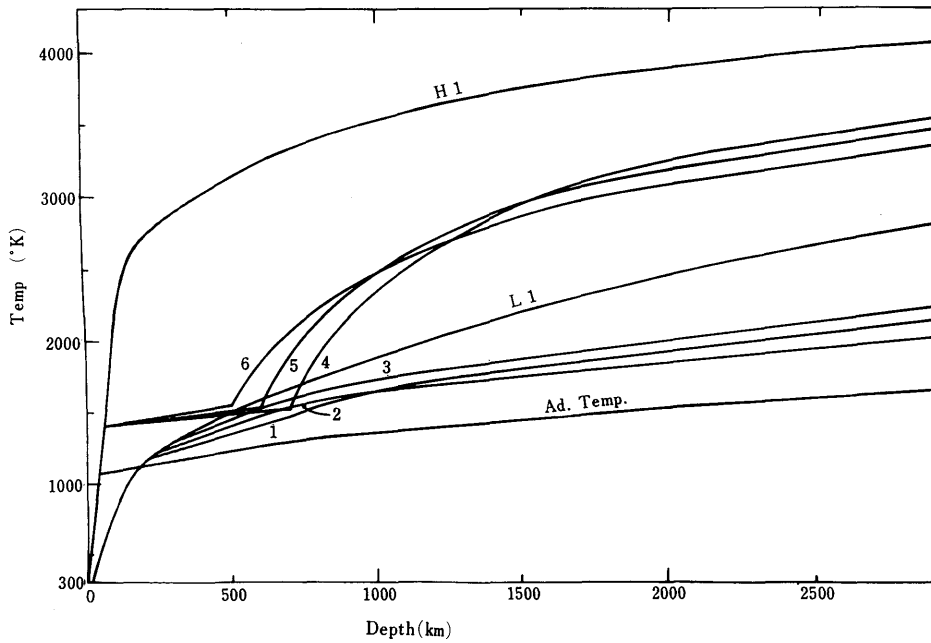


Fig. 1 Steady convection solutions $T(r)$ for hot (H1, Iriyama,⁵⁾ his model RC-U2) and cold (L1) models.

H1 : 4, 5 and 6 ; Upper boundary of the upper mantle convection at depth 60 km.

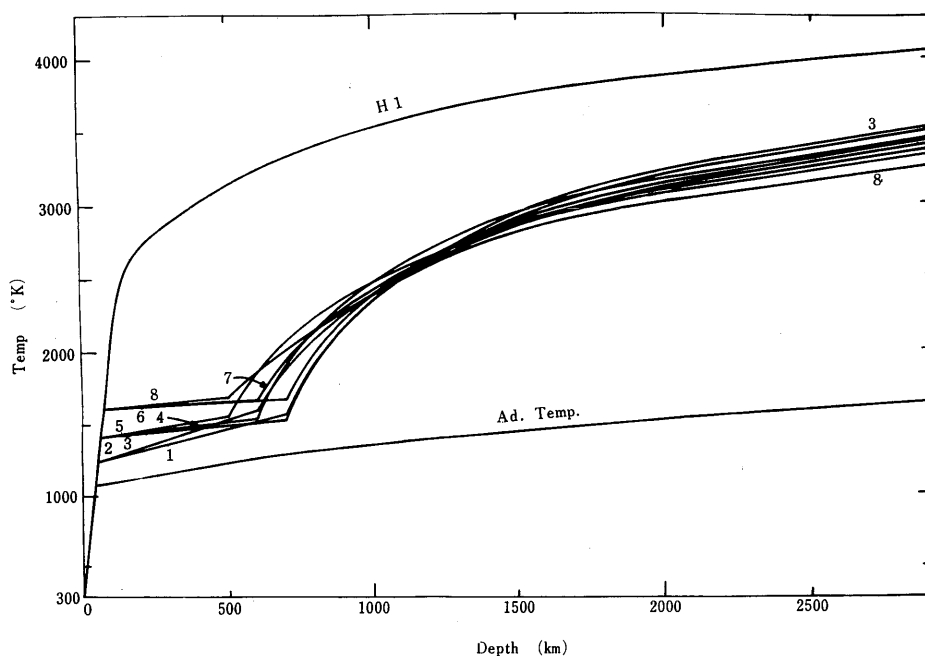
Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

L1 : 1, 2 and 3 ; Upper boundary of the upper mantle convection at depth 210, 230 and 290 km, respectively.

Ad. Temp. ; Adiabatic temperature distribution by Verhoogen.⁶⁾

Table 2 Data for cold model (model L1)

Model	UB , km	d , km	T_{UB} , °K	T_{LB} , °K	η , poise	Ra	Nu	T_m , °K
L1-500'	250 500	250 2400	1228 1505	1505 2808	2.27×10^{22}	1.15×10^8		
L1-500	290 500	210 2400	1280 1505	1505 2808	5.09×10^{21} 3.18×10^{25}	2.33×10^8 3.03×10^8	1.40 1.79	1448 2232
L1-500''	330 500	170 2400	1330 1505	1505 2808	1.35×10^{21} 6.89×10^{25}	3.42×10^8 1.45×10^8	1.87	1423
L1-600'	220 600	380 2300	1183 1593	1593 2808	9.21×10^{22}	1.55×10^8		
L1-600	230 600	370 2300	1200 1593	1593 2808	5.36×10^{22} 1.27×10^{25}	2.31×10^8 6.70×10^8	1.33 2.84	1480 2021
L1-600''	240 600	360 2300	1213 1593	1593 2808	3.58×10^{22} 6.47×10^{25}	3.04×10^8 1.43×10^8	1.79	1425
L1-700'	200 700	500 2200	1150 1673	1673 2808	2.76×10^{23}	1.56×10^8		
L1-700	210 700	490 2200	1170 1673	1673 2808	1.41×10^{23} 1.51×10^{25}	2.70×10^8 5.12×10^8	1.66 2.43	1473 2139
L1-700''	220 700	480 2200	1183 1673	1673 2808	9.21×10^{22}	3.74×10^8	2.09	1417

**Fig. 2** Steady convection solutions $T(r)$ for model H1.

1, 2, 3, 4, 5, 6, 7 and 8 ; Upper boundary of the upper mantle convection at depth 50, 50, 60, 60, 60, 70, 70, and 70 km, respectively. Upper boundary of the lower mantle convection at depth 700, 600, 700, 600, 500, 700, 600 and 500 km, respectively.

upper mantle. In this case, the temperature at the lower boundary of lower mantle convection changes by some 300 degrees.

4. If the convection is present, then the temperature gradient within the upper mantle differs very little from the adiabatic gradient. The temperature at the core-

mantle boundary is about 3300°K for model H1 and 2100°K for model L1. The result is close to that given by Lyustikh⁷⁾ by a quantitative study.

5. The upper mantle of the earth is considerably less thermally stable than the lower mantle.

6. If the temperature at the bottom of the upper mantle convection cell is low, then the temperature of the lower mantle convection system is high. This results from the fact that the viscosity becomes larger or the Rayleigh number smaller as the temperature becomes lower.

Acknowledgements

The author is indebted to Prof. Yasuo Shimazu (Nagoya University) for valuable advice and to Mr. Kazuo Fukuta for technical assistance. Further, the author wishes to thank Prof. Makisaburo Ogasawara for his valuable comments.

This paper was presented mainly to the annual meeting of the Seismological Society of Japan, May 1971.

This study was supported in part by a grant-in-aid for Special Scientific Research, Chubu Institute of Technology.

References

- 1) Cox, J.P., & Giuli, R.T. (1968). "Principles of Stellar Structure, Vol. 1; Physical Principles", Gordon & Breach, New York.
- 2) Tozer, D.C. (1967). In "The Earth's Mantle" (T.F. Gaskell, ed.), p. 325, Academic Press, London.
- 3) Knopoff, L. (1964). *Rev. Geophys.* **2**, 89-122.
- 4) Silveston, P.L. (1958). *Forch. Ing. Wes.* **24**, 29-32, 59-69.
- 5) Iriyama, J. (1970). *J. Phys. Earth.* **18**, 295-311.
- 6) Verhoogen, J. (1951). *Trans. Am. Geophys. Un.* **32**, 41-43.
- 7) Lyustikh, E. N. (1969). In "The Earth's Crust and Upper Mantle" (P.J. Hart, ed.), p. 689, American Geophysical Union, Washington, D.C.