

Nucleon Electromagnetic Mass Difference and Inelastic Electron Scattering

Norikazu MORISHITA*

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We consider the p - n mass difference on the basis of a dispersion theory. We evaluate the inelastic and Regge contributions by using (1) the $T_1(q^2, \nu)$ dispersion relation subtracted at $\nu=0$, (2) predictions of the vector-meson dominance model for inelastic electron-nucleon scattering, and (3) total photoproduction cross sections of the proton and neutron given by simple Regge pole fits of the experimental data. Thus we find $\delta M^{\text{inel}}(p-n) + \delta M^{\text{Regge}}(p-n) \simeq 0.4$ MeV.

§ 1. Introduction

Recently a series of interesting experiments¹⁾ of inelastic lepton-nucleon scattering has been carried out and the results have determined the inelastic electromagnetic nucleon form factors. Several interesting theoretical models have also been proposed to explain the feature revealed by these experiments. The typical examples are the vector-meson dominance (VMD) model,²⁾ the parton model,^{2a)} and the Regge pole model.³⁾

In this paper we consider the p - n mass difference starting with the Cottingham formulation,⁴⁾ which relates the mass difference to the experimental differential cross sections for elastic and inelastic electron-nucleon scattering. Especially we estimate the inelastic contribution to the p - n mass difference. We then use the VMD predictions for the form factors W_1 and W_2 , and total photoproduction cross sections of the proton and neutron given by simple Regge pole fits of the recent experimental data. For any $\Delta I=1$ mass difference Harrari⁵⁾ pointed out on the basis of a Regge pole model that we should introduce a subtraction for the amplitude $T_1(q^2, \nu)$ since its behavior at high values of ν is dominated by the A_2 Regge pole. Therefore, using the $T_1(q^2, \nu)$ dispersion relation once-subtracted at $\nu=0$, we also estimate the additional contribution arising from this subtraction.

§ 2. Electromagnetic self-energy of the nucleon

To first order in the fine structure constant α the electromagnetic self-energy of a nucleon has been expressed as

$$\delta M = -\frac{\alpha}{4\pi^2} \int_0^\infty \frac{dq^2}{q^2} \int_{-q}^q d\nu (q^2 - \nu^2)^{1/2} \left[3T_1(q^2, i\nu) - \frac{q^2 - \nu^2}{q^2} T_2(q^2, i\nu) \right], \quad (1)$$

where $T_1(q^2, \nu)$ and $T_2(q^2, \nu)$ are related to

$$T_{\mu\nu}(q^2, \nu) = T_1(q^2, \nu) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{T_2(q^2, \nu)}{M^2} \left(P_\mu + \frac{M\nu}{q^2} q_\mu \right) \left(P_\nu + \frac{M\nu}{q^2} q_\nu \right), \quad (2)$$

which is the forward Compton amplitude for the scattering of a virtual photon with mass $-q^2$ and laboratory energy $q^0 = \nu$ from a nucleon of mass M and momentum P .

*Department of Engineering Physics.

For $T_1(q^2, \nu)$ and $T_2(q^2, \nu)$ we consider fixed- q^2 dispersion relations in the variable ν . Following to the Harari's argument, we assume that $T_1(q^2, \nu)$ obeys a once-subtracted dispersion relation⁵⁾ and $T_2(q^2, \nu)$ an unsubtracted one as follows

$$T_1(q^2, \nu) = T_1(q^2, 0) + \frac{\pi}{\alpha} \frac{16M^3\nu^2}{q^4 - 4M^2\nu^2} [f_1(q^2) + \frac{q^2}{4M^2} f_2(q^2)] \\ + \frac{2\nu^2}{\pi} \int_{\nu_t}^{\infty} \frac{\text{Im} T_1(q^2, \nu')}{\nu'^2 - \nu^2} \frac{d\nu'}{\nu'}, \quad (3)$$

$$T_2(q^2, \nu) = \frac{\pi}{\alpha} \frac{4Mq^4}{q^4 - 4M^2\nu^2} f_2(q^2) + \frac{2}{\pi} \int_{\nu_t}^{\infty} \frac{\text{Im} T_2(q^2, \nu')}{\nu'^2 - \nu^2} \nu' d\nu', \quad (4)$$

where $f_1(q^2)$ and $f_2(q^2)$ are certain combinations of the electric and magnetic form factors of the nucleon and ν_t is the inelastic threshold. In order to determine the subtraction term we use the conventional Regge pole model, in which the asymptotic form of $T_1(q^2, \nu)$ is expressed by the leading trajectory A_2 as

$$T_1^R(q^2, \nu) = \beta(q^2) \frac{1 + \exp[-i\pi\alpha(0)]}{\sin \pi\alpha(0)} \nu^{\alpha(0)}, \quad (5)$$

where $\alpha(0) = \alpha_{A_2}(0)$ is the $t=0$ intercept of the A_2 trajectory, which is about $1/2 \sim 1/3$,⁶⁾ and $\beta(q^2)$ is its residue, of which the explicit form will be given in the next section. In order to simplify our calculation we take the value

$$\alpha(0) = 1/2. \quad (6)$$

We also write an unsubtracted dispersion relation for $T_1(q^2, \nu) - T_1^R(q^2, \nu)$ and obtain

$$T_1(q^2, \nu) - T_1^R(q^2, \nu) = \frac{\pi}{\alpha} \frac{16M^3q^2}{q^4 - 4M^2\nu^2} [f_1(q^2) + \frac{q^2}{4M^2} f_2(q^2)] \\ + \frac{2}{\pi} \int_{\nu_t}^{N(q^2)} \frac{\text{Im} T_1(q^2, \nu')}{\nu'^2 - \nu^2} \nu' d\nu' + \frac{2}{\pi} \beta(q^2) \frac{N^{\alpha(0)}(q^2)}{\alpha(0)}, \quad (7)$$

where $N(q^2)$ defines the beginning of the Regge asymptotic region. By putting $\nu=0$ in (7) we find the subtraction term $T_1(q^2, 0)$. Thus we can write down as

$$T_1(q^2, \nu) = \frac{\pi}{\alpha} \frac{4Mq^4}{q^4 - 4M^2\nu^2} [f_1(q^2) + \frac{\nu^2}{q^2} f_2(q^2)] + \frac{2}{\pi} \beta(q^2) \frac{N^{\alpha(0)}(q^2)}{\alpha(0)} \\ + \frac{2}{\pi} \int_{\nu_t}^{N(q^2)} \frac{\text{Im} T_1(q^2, \nu')}{\nu'^2 - \nu^2} \nu' d\nu' + \frac{2\nu^2}{\pi} \beta(q^2) \int_0^{\infty} \frac{\nu'^{\alpha(0)}}{\nu'^2 - \nu^2} \frac{d\nu'}{\nu'}. \quad (8)$$

After some analysis we obtain

$$\delta M = \delta M^{\text{el}} + \delta M_1^{\text{inel}} + \delta M_1^{\text{Regge}} + \delta M_2^{\text{inel}}, \quad (9)$$

where δM^{el} and δM_1^{inel} are the contributions of terms involving the elastic and inelastic subtraction terms obtained from (7) with $\nu=0$, respectively, therefore δM^{el} becomes the Cottingham's expression⁴⁾ which follows from the unsubtracted dispersion relation. The term $\delta M_1^{\text{Regge}}$, which is the subtraction contribution except the elastic and inelastic subtraction contributions, is given by

$$\delta M_1^{\text{Regge}} = -\frac{3\alpha}{2\pi^2} \int_0^{\infty} \beta(q^2) N^{1/2}(q^2) dq^2, \quad (10)$$

where $\alpha(0)$ has been replaced with $1/2$. The inelastic contributions are given by

$$\delta M_1^{\text{inel}} = -\frac{3\alpha}{2\pi} \int_0^{\infty} \frac{dq^2}{q^2} \int_{\nu_t}^{N(q^2)} \left(-1 + \sqrt{1 + \frac{q^2}{\nu'^2}} \right) W_1(q^2, \nu) \nu d\nu \\ - \frac{\alpha}{8\pi^2} \int_0^{\infty} q^2 \beta(q^2) N^{-3/2}(q^2) dq^2, \quad (11)$$

$$\delta M_2^{\text{inel}} = -\frac{\alpha}{2\pi} \int_0^\infty \frac{dq^2}{q^2} \int_{\nu_i}^\infty \left[\left(1 + \frac{\nu^2}{q^2} \right) \left(-1 + \sqrt{1 + \frac{q^2}{\nu^2}} \right) - \frac{1}{2} \right] W_2(q^2, \nu) \nu d\nu, \quad (12)$$

where we have used the Drell-Walecka form factors $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ ⁷⁾ which are related to the absorptive parts of $T_1(q^2, \nu)$ and $T_2(q^2, \nu)$ as

$$W_i(q^2, \nu) = \pi^{-1} \text{Im} T_i(q^2, \nu) \quad (i=1, 2). \quad (13)$$

§ 3. Estimation of the inelastic and Regge contributions

In calculating (10), (11), and (12) we need the explicit forms of $W_1(q^2, \nu)$, $W_2(q^2, \nu)$, and $\beta(q^2)$, and so we now consider these functions. Recently inelastic electron-nucleon scattering experiments have been carried out widely, and the results have determined the inelastic electromagnetic form factors. At present there exists no complete theory by which we can calculate the form factors. However, several interesting theoretical proposals have been made for the form factors. As one of these proposals let us take the VMD model.

The form factors $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ are written as

$$W_1(q^2, \nu) = \frac{K}{4\pi^2 \alpha} \sigma_T(q^2, \nu),$$

$$W_2(q^2, \nu) = \frac{K}{4\pi^2 \alpha} \frac{q^2}{q^2 + \nu^2} (1 + R) \sigma_T(q^2, \nu), \quad (14)$$

where $K = \nu - q^2/2M$, $R = \sigma_L/\sigma_T$, and σ_T and σ_L are the transverse and longitudinal virtual photo-nucleon cross sections, respectively. The VMD model predicts that σ_T in inelastic electron-nucleon scattering is given by²⁾

$$\sigma_T(q^2, \nu) = \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right)^2 \left(\frac{e}{f_\rho} \right)^2 \sigma_{\rho N}(K), \quad (15)$$

where $(e/f_\rho)^2 \sigma_{\rho N}(K)$ is simply replaced by the total photoproduction cross section $\sigma_{\gamma N}(K)$. According to the Regge pole model let us consider that $\sigma_{\gamma N}(K)$ consists of a constant contribution due to Pomeranchukon t -channel exchange and another contribution to go to zero as $K \rightarrow \infty$ due to the $I=1$ trajectory exchange. The difference $\sigma_{\gamma p}(K) - \sigma_{\gamma n}(K)$ consists of the contribution only due to the $I=1$ trajectory which is dominated by the A_2 trajectory. We use the value for $\sigma_{\gamma p}(K) - \sigma_{\gamma n}(K)$ given by a simple Regge pole fit⁸⁾ of the experimental data,

$$K[\sigma_{\gamma p}(K) - \sigma_{\gamma n}(K)] = 24.6 K^{\alpha(0)} \text{ (GeV} \cdot \mu\text{b)}. \quad (16)$$

With the above assumptions we obtain comparatively simple forms

$$W_1^p(q^2, \nu) - W_1^n(q^2, \nu) = \frac{63.3 \times 10^{-3}}{4\pi^2 \alpha} \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right)^2 K^{1/2} \text{ (GeV}^{-1}), \quad (17)$$

$$W_2^p(q^2, \nu) - W_2^n(q^2, \nu) = \frac{63.3 \times 10^{-3}}{4\pi^2 \alpha} (1 + R) \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right)^2 \frac{q^2}{q^2 + \nu^2} K^{1/2} \text{ (GeV}^{-1}). \quad (18)$$

Comparing (17) with (5) we also obtain the explicit form of $\beta(q^2)$

$$\beta^p(q^2) - \beta^n(q^2) = -\frac{63.3 \times 10^{-3}}{4\pi \alpha} \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right)^2 \text{ (GeV}^{-3/2}). \quad (19)$$

Using (17)~(19) we can carried out the calculations of (10)~(12). Since δM_1^{inel} and $\delta M_1^{\text{Regge}}$ depend on the Regge asymptotic cutoff $N(q^2)$, we reasonably consider that $N(q^2)$ is greater of the order of 2 GeV than the elastic threshold $q^2/2M$. The evaluations of $\delta M_1^{\text{inel}}(p-n)$ and $\delta M_1^{\text{Regge}}(p-n)$ are displayed in Table I.

Table I. Calculations of $\delta M_1^{\text{inel}}(p-n)$ and $\delta M_1^{\text{Regge}}(p-n)$ with $N(q^2)=c+q^2/2M$.

	$\delta M_1^{\text{inel}}(p-n)$ (MeV)	$\delta M_1^{\text{Regge}}(p-n)$ (MeV)	Total (MeV)
$c=1.5$ GeV	-0.32	0.71	0.39
2.0 GeV	-0.37	0.78	0.41
2.5 GeV	-0.41	0.84	0.43

The contribution $\delta M_2^{\text{inel}}(p-n)$ depends on the ratio $R=\sigma_L/\sigma_T$ which has measured experimentally only for very few values of q^2 and K . For values of $K/q^2 < 3 \text{ GeV}^{-1}$, the results indicate $R \leq 0.5$.⁹⁾ And R may smoothly vanish for $q^2 \rightarrow 0$ since σ_L must vanish there. For these reasons we may assume that R is a constant of the order of 0~1, which gives us

$$\delta M_2^{\text{inel}}(p-n) \sim 0.01 \text{ MeV}. \quad (20)$$

Thus we may suppose that $\delta M_2^{\text{inel}}(p-n)$ is fairly small for unknown R . We then obtain the final result,

$$\delta M_1^{\text{inel}}(p-n) + \delta M_1^{\text{Regge}}(p-n) + \delta M_2^{\text{inel}}(p-n) \simeq 0.4 \text{ MeV}. \quad (21)$$

§ 4. Closing remarks

As is well known, the elastic term in (8) yields $\delta M^{\text{el}}(p-n) \simeq 0.8 \text{ MeV}$,¹⁰⁾ while the observed $p-n$ mass difference is -1.29 MeV . We can see from Table I that the inelastic contribution gives a right sign but the Regge contribution of the subtraction term has a positive sign compensating the inelastic contribution. Finally a small positive contribution, $\delta M_1^{\text{inel}}(p-n) + \delta M_1^{\text{Regge}}(p-n) + \delta M_2^{\text{inel}}(p-n) \simeq 0.4 \text{ MeV}$, aggravates the situation. Thus it seems impossible to understand the $p-n$ mass difference within the framework of dispersion theory as long as we rely on the way investigated here. The VMD model is supposed to be best when q^2 is small and ν is large. Therefore, it is not clear that the contribution from the high- q^2 region has reasonably been estimated. We may also need to consider the presence of $I=1$ and $J=1$ fixed pole¹¹⁾ together with the usual Regge poles. Lastly we only refer to an interesting fact that several authors^{11,12)} have attempted to calculate the Regge contribution in various procedures, but their predicted results do not always give a same sign.

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References

- 1) E. D. Bloom *et al.*, Phys. Rev. Letters **23** (1969), 930; B. T. Dieterle *et al.*, *ibid.* **23** (1969), 1187; I. Budagov *et al.*, Phys. Letters **30B** (1969), 364.
- 2) J. J. Sakurai, Phys. Rev. Letters **22** (1969), 981; C. F. Cho, G. J. Gounaris, and J. J. Sakurai, Phys. Rev. **186** (1969), 1734.
- 2a) R. P. Feynman, Phys. Rev. Letters **25** (1969), 1218; J. D. Bjorken and E. Paschos, Phys. Rev. **185** (1969), 1975.
- 3) H. Pagels, Phys. Rev. D **3** (1971), 1217; J. W. Moffat and V. G. Snell, *ibid.* **3** (1971), 1217.

- 2848.
- 4) W. Cottingham, Ann. Phys. (N. Y.) **25** (1963), 424.
 - 5) H. Harari, Phys. Rev. Letters **17** (1966), 1303.
 - 6) See for instance, M. N. Focacci *et al.*, Phys. Rev. Letters **17** (1966), 890.
 - 7) These form factors are related to the functions $h_{1,2}(q^2, \nu)$ introduced, for example, in Ref. 4; $\alpha W_1(q^2, \nu) = q^2 h_1(q^2, \nu) + \nu^2 h_2(q^2, \nu)$ and $\alpha W_2(q^2, \nu) = q^2 h_2(q^2, \nu)$.
 - 8) W. P. Hesse *et al.*, Phys. Rev. Letters **25** (1970), 613.
 - 9) R. Taylor, *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p.251.
 - 10) M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2** (1959), 7; J. H. Wojtaszek, R. E. Marshak, and Riazuddin, Phys. Rev. **136** (1964), B1053; S. Coleman and S. Glashow, *ibid.* **134** (1964), B671.
 - 11) R. Chanda, Phys. Rev. **188** (1969), 1988; M. Elitzur and H. Harari, Ann. Phys. (N. Y.) **56** (1970), 81.
 - 12) Y. Hara, Nucl. Phys. **B8** (1968), 441.