

Thermal History of the Earth and the Formation of Its Core ; Part II

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The hypothesis of a rapid growth in the Earth's core is tested from the thermal history of the Earth. Calculation shows that the conversion of gravitational to thermal energy from a uniform to the present differentiated Earth is 1.68×10^{38} ergs.

Earth models in which the core was substantially complete at 4500 My ago or 3000 My ago within a short period less than 100 My give the observed heat flow.

1. Introduction

The separation of the core is the principal event of the Earth's evolution. The reconstruction of the Earth from a uniform, metal-silicate phase mixture to the present state has released substantial amounts of gravitational energy. We should not ignore the gravitational energy when calculating theoretical models of thermal history of the Earth containing the iron core. However, reports of this type of study have apparently not been published to date.

The time-scale of the core accumulation is uncertain. Stacey¹⁾, Birch^{2),3)}, Shimazu⁴⁾, Tozer⁵⁾, and Fujii and Uyeda⁶⁾ argue that the core was substantially complete at an early stage in the Earth's history within a short period less than 100 My. The purpose of this paper is to study the thermal history of the Earth assuming a rapid formation of the core resulting from the differentiation of the mantle.

The temperature of the surface is always adopted as equal to 273°K. The age of the Earth is set equal to 4.5×10^9 years. We assume that in the original Earth the metallic iron is preexisted and distributed among silicate phase. The concentration of radioactive elements throughout the Earth until core formation commences is assumed to be equal to that of chondritic meteorites. At the original state of uniform chondritic composition, we put $U=1 \times 10^{-8}$ g/g, $K^{40}=8 \times 10^{-8}$ g/g (reduced to the present values), and $Th/U=4$. After separation of the core, the abundance of U , Th , and K^{40} in the silicate mantle is changed to satisfy the mass conservation, preserving the ratio of Th/U .

2. Analysis and Model of Rapid Core Formation

In this section processes of core formation are analyzed and are classified.
(2-1). *Initial cold Earth (RC)* :

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The commencement of core differentiation would be determined by melting of the iron and by a reduction in the viscosity of silicate phase in the mantle, as pointed out by Tozer⁵⁾, and Uyeda⁷⁾. The formation of the core would not proceed until the mantle is, by radioactive heating, to become soft enough to permit the iron to fall. We may take the temperature dependence of the mantle viscosity to be of the form $\exp(E/kT)$ where E is an activation energy and k the Boltzmann's constant. Hence, as Tozer⁵⁾ has argued, the process of core formation releases a lot of energy which will cause a further fall in viscosity, so that this process will quickly establish itself or be catastrophic. The rate of the gravitational energy release will be itself dependent on the rate of core formation in a manner which encourages a catastrophic change.

From this point of view the time scale of the core accumulation would be a geologically short time less than 100 My. Now, if we assume an initially cold Earth, the core is substantially complete at the time between 1500 to 1600 My after the end of the accretion process of the Earth. The rapid growth of the core provides a source of energy fast released at the time between 1500 and 1600 My. The initial temperature distribution $T_0(r)$ in this case is given in Figure 1, which is reproduced from that calculated by Lubimova⁸⁾. This core formation process is referred to as model RC.

(2-2). *Initial hot Earth (RH) :*

From geochronological data, the Earth has an age of about 4500 My. Shimazu⁴⁾ argues that this age refers to the time when the upper mantle and the crust can be regarded as a closed system with characteristic U/Pb, and the value of 4500 My is regarded as the age of the upper mantle. Shimazu⁴⁾ has pointed out that the separation of the core affects the U/Pb ratio of the silicate mantle, for the metal carries down significant amounts of Pb but not U. Hence, if the mantle-core separation is continued significantly during the stage of a closed system, the consistent value of 4500 My for the age of mantle cannot be obtained from rocks differentiated from the mantle at different ages. Hence, Shimazu⁴⁾ argues emphatically that the Earth had separated into the core and the mantle at or before 4500 My ago.

From this point of view it is necessary to assume that the core was substantially complete at the time between 0.0 to 1.0×10^8 years. The rapid growth of the core provides a source of energy fast released at the time between 0.0 to 1.0×10^8 years. The initial temperature distribution $T_0(r)$ in this case is given in Figure 2. This core formation process is referred to as model RH.

3. Theory

Here various basic formulas are analyzed as follows :

(3-1). *Loss of gravitational potential energy.*

For an undifferentiated and the present differentiated Earth, we calculate the gravitational potential energies. The loss of gravitational energy is the difference of the two large quantities. As a result of the rapid growth of the core, we shall assume that the gravitational energy is rapidly emitted over the whole body of the Earth within a short period less than 100 My.

To construct of a density model for the undifferentiated Earth, we assume an equation of state of the form (MacDonald⁹)

$$d\rho/dr = \rho_0^3 g / (\alpha \rho^2), \quad (1)$$

where ρ_0 =surface density and α =ratio of incompressibility to density.

In the undifferentiated Earth ρ_0 and α are taken to be 4.09 g/cm³ and 29 (km/sec)² respectively. The boundary conditions at $r=0$ (center of the Earth) and R (surface) are as follows :

$$r=0 : dP/dr=0, \quad (2)$$

$$g=0, \quad (3)$$

$$r=R : dP/dr = \rho(R)g(R). \quad (4)$$

The gravitational potential energy of the Earth (negative), Ω , is expressible in a Protean variety of forms (Birch²) :

$$\Omega = -4\pi \int_0^R g \rho r^3 dr = -12\pi \int_0^R P r^2 dr. \quad (5)$$

(3-2). Heat transfer

The equation governing the transfer of heat is given by

$$\partial T / \partial t = (1/\rho C_p) [\nabla \cdot (K \nabla T) + A + G], \quad (6)$$

where K =total thermal conductivity,

G =rate of heat production by gravitation,

A =rate of heat production by radioactivity,

C_p =specific heat=1.26 joules/g-deg.

We shall take the thermal conductivity in the mantle to be of the form

$$K = K_{\text{photon}} + K_{\text{phonon}}. \quad (7)$$

The photon conductivity (K_{photon}) is given by

$$K_{\text{photon}} = 16n^2 \beta T^3 / (3\varepsilon), \quad (8)$$

where n^2 =refractive index=3, ε =opacity=10 cm⁻¹, and β stands for the Stefan-Boltzmann constant.

The phonon conductivity (K_{phonon}) is given by

$$K_{\text{phonon}} = B / T, \quad (9)$$

where the constant B is determined from the condition that on the Earth's surface (where $T=277^\circ\text{K}$) the thermal conductivity K_{phonon} should be equal to the conductivity of the crustal rocks $K=0.0334$ joules/cm-sec-deg (Levin¹⁰).

The thermal conductivity of the core is assumed to be due to free electron or 0.418 joules/cm-sec-deg.

4. Results

Rapid formation of the core

(4-1). Initial cold Earth (RC) :

In model RC we investigate the thermal history of an initially cold Earth with consideration of the energy emitted in the rapid growth of the core. The densities for the initial, undifferentiated Earth are listed in Table 1. As compared with previous paper (Iriyama¹¹), the present paper has improved in the accuracy of density calculation.

For the present Earth (Bullard¹²⁾), the gravitational potential energy is -2.488×10^{39} ergs; for the undifferentiated Earth, -2.290×10^{39} ergs. The total loss of the gravitational energy due to core-mantle separation becomes 1.98×10^{38} ergs. However, the lost gravitational energy is not completely converted to heat. After Urey¹³⁾ and Birch³⁾ we assume that some 15% of the loss of gravitational energy remains as strain energy in the Earth's interior. Therefore the net energy change in reconstruction of the Earth is 1.68×10^{38} ergs. This is 1.5 times greater than the total amounts of heat 1.10×10^{38} ergs liberated by radioactivity in the chondritic Earth (see Section 1). It is assumed that the gravitational energy had been rapidly emitted over the whole body of the Earth at the time between 1.5 to 1.6×10^9 years.

In the present calculations we shall use three models about the concentration of radioactive elements in the mantle. In one model, as a result of the rapid growth of the core at $t = 1.5 \times 10^9$ years, we assume that the radioactivity had been rapidly concentrated in the upper 600 km of the Earth at the time between 1.5 to 1.6×10^9 years. In a second model we assume that the radioactivity had been rapidly concentrated in the upper 200 km at that time. Finally, for the sake of comparison, it is assumed that the radioactivity had been rapidly concentrated in the whole mantle at that time. These three models are referred to as model RC-U6, RC-U2, and RC-M respectively.

The development of temperature as a function of the time within model RC-U6 is shown in Figure 1. It is seen that at all depths there was a sharp increase in temperature at the time between 1.55 to 1.65×10^9 years. After the very rapid rise in temperature, the outer layers of the Earth cool rapidly, while at all depths greater than 1400 km the temperature does not greatly change with time. It is clearly seen that the gravitational energy has dominated the thermal history of the Earth.

In models RC-U6, RC-U2, and RC-M the present temperature distributions are listed in Table 2. In model RC-U2 the present temperature is higher than that of RC-U6 in the uppermost layer.

Figure 2 shows the history of surface heat flow for RC models. In all these models,

Table 1. Density and pressure in the undifferentiated Earth.

Radius, km	Density, g/cm ³	Pressure, Mb
0	6.96	2.150
300	6.95	2.139
600	6.93	2.112
900	6.90	2.074
1200	6.87	2.024
1500	6.82	1.962
1800	6.77	1.890
2100	6.70	1.808
2400	6.63	1.715
2700	6.54	1.613
3000	6.45	1.503
3300	6.34	1.385
3600	6.22	1.260
3900	6.09	1.129
4200	5.94	0.993
4500	5.77	0.854
4800	5.58	0.712
5100	5.38	0.569
5400	5.14	0.428
5700	4.87	0.289
6000	4.56	0.155
6300	4.19	0.029
6371	4.09	0.000

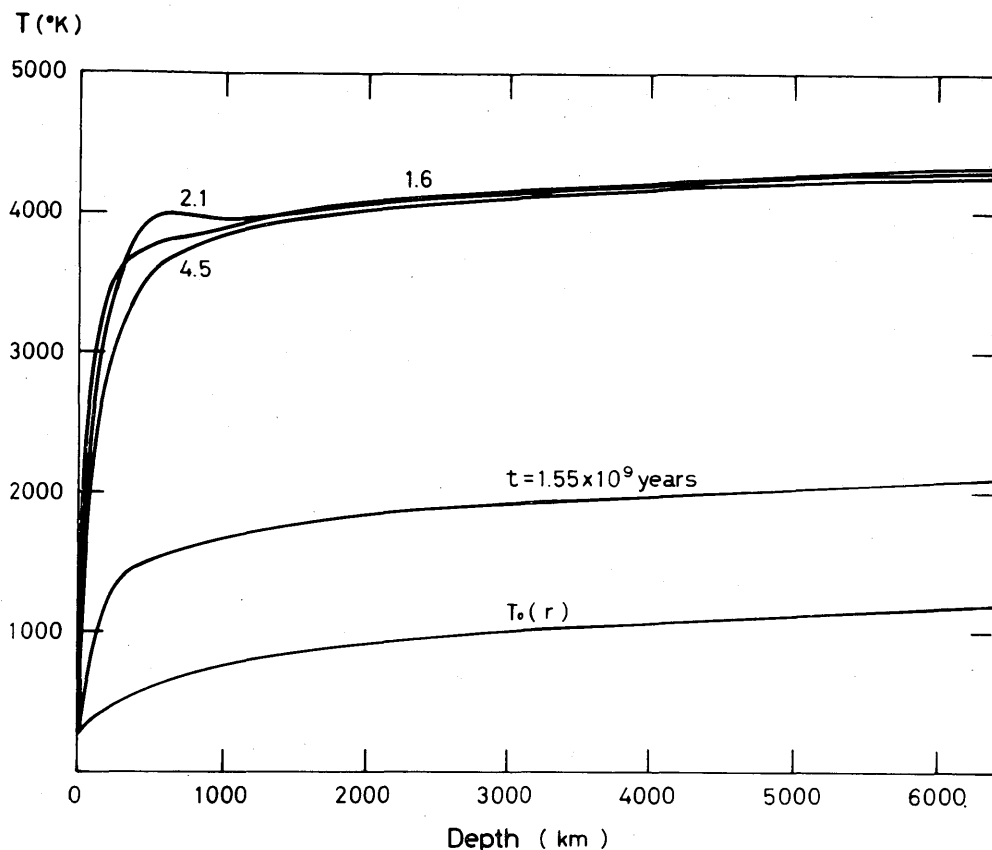


Fig. 1 The development of the temperature distribution within model RC-U6.

Table 2. Present temperature distributions for RC models.

Depth, km	Model RC-U6 T, °K	Model RC-U2 T, °K	Model RC-M T, °K
100	2150	2160	1810
200	2820	2720	2440
400	3390	3030	3050
600	3640	3260	3410
800	3740	3420	3660
1000	3810	3540	3850
1500	3930	3760	4150
2000	4000	3910	4320
2900	4100	4070	4370
4000	4180	4180	4250
5000	4230	4230	4240
6370	4260	4260	4260

at $t = 1.6 \times 10^9$ years, there was a sharp increase in heat flow followed by a rapid decrease. The maximum value of the heat flow took place at $t = 1.63 \times 10^9$ years, i.e., 2.87×10^9 years ago,

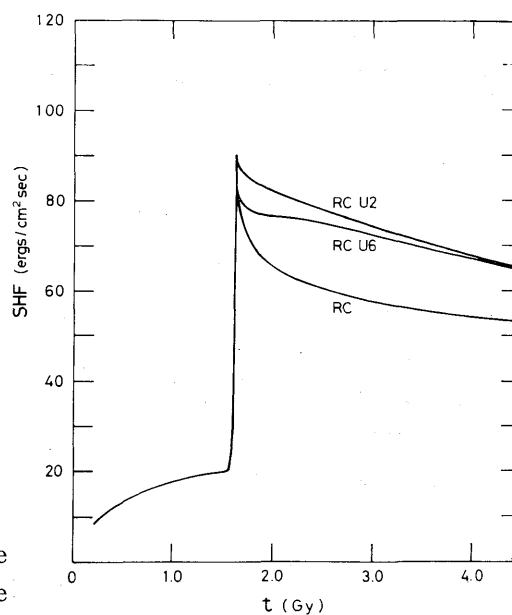


Fig. 2 Variation of surface heat flow with time for RC models.

which occurred about 1.3×10^8 years after the differentiation of the core. For model RC-U6, the present surface heat flow is $62.8 \text{ ergs/cm}^2\text{-sec}$; for model RC-U2, $63.1 \text{ ergs/cm}^2\text{-sec}$; for model RC-M, $51.4 \text{ ergs/cm}^2\text{-sec}$, which are in agreement with the observed heat flow (Uyeda¹⁴).

(4-2). *Initial hot Earth (RH)* :

In model RH we investigate the thermal history of an initially hot Earth with consideration of the energy emitted in the rapid growth of the core. It is assumed that the gravitational energy had been rapidly emitted over the whole body of the Earth at the time between 0.0 to 1.0×10^8 years.

In the present calculations we shall use three models about the concentration of radioactive elements in the mantle. In one model, as a result of the rapid growth of the core at $t=0.0$, we assume that the radioactivity had been rapidly concentrated in the upper 600 km of the Earth at the time between 0.0 to 1.0×10^8 years. In a second model we assume that the radioactivity had been rapidly concentrated in the upper 200 km at that time. Finally, we assume that the radioactivity had been rapidly concentrated in the whole mantle at that time. These three models are referred to as

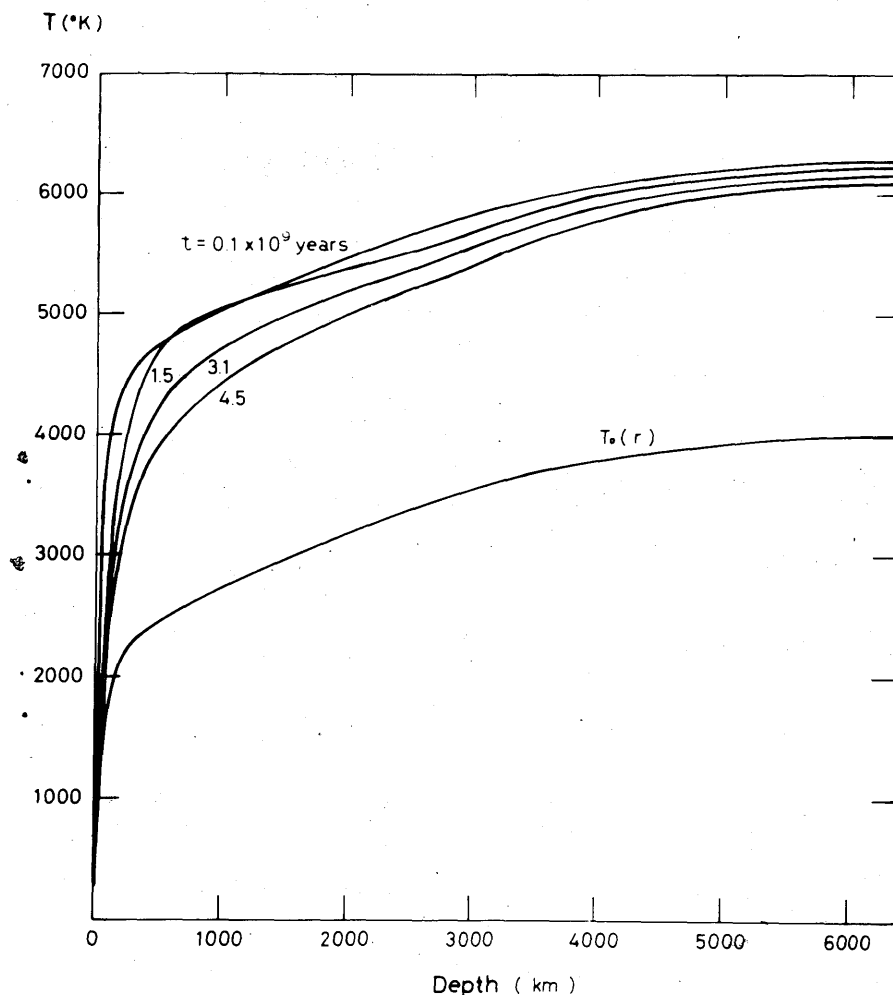


Fig. 3 The development of the temperature distribution within model RH-U6.

model RH-U6, RH-U2, and RH-M respectively.

The development of temperature as a function of time within model RH-U6 is shown in Figure 3. It is seen that at all depths there was sharp increase in temperature at the time between 0.2 to 1.1×10^8 years. This brief temperature rise at the very beginning soon ceased and cooling commenced. At $t > 1.1 \times 10^8$ years the outer layers of the Earth cool rapidly, while at all depths greater than 3000 km the temperature decreases slowly with time until the present time.

In models RH-U6, RH-U2, and RH-M the present temperature distributions are listed in Table 3. In model RH-U2 the present temperature is lower than that in model RH-

Table 3. Present temperature distributions for RH models.

Depth, km	Model RH-U6 T, °K	Model RH-U2 T, °K	Model RH-M T, °K
100	2330	2290	2140
200	3160	2920	2860
400	3710	3370	3560
600	4050	3690	4000
800	4250	3930	4310
1000	4420	4130	4550
1500	4740	4510	5000
2000	4980	4810	5320
2900	5370	5250	5670
4000	5800	5760	5960
5000	6000	5990	6060
6370	6100	6100	6120

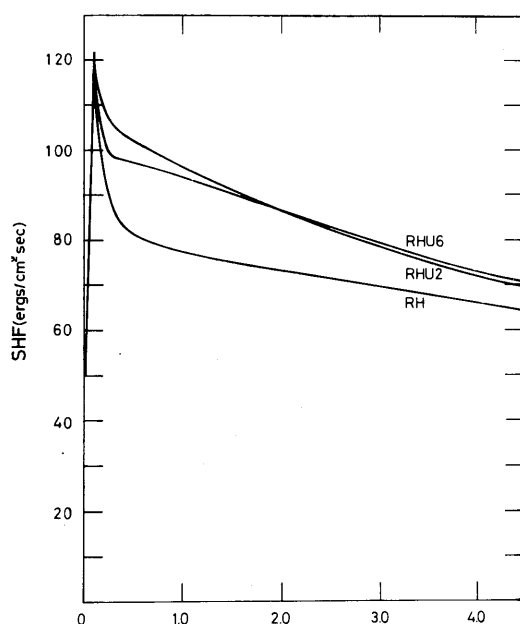


Fig. 4 Variation of surface heat flow with time for RH models.

U6 at all depths.

Figure 4 shows the history of surface heat flow for RH models. In all these models, at the very beginning, there was a sharp increase in surface heat flow followed by a rapid decrease.

The maximum value of the heat flow took place at $t = 1.1 \times 10^8$ years, i.e., 4.39×10^9 years ago, which occurred some 1.1×10^8 years after the differentiation of the core. For model RH-U6, the present surface heat flow is 68.7 ergs/cm²-sec ; RH-U2, 67.4 ergs/cm²-sec ; for model RH-M, 62.4 ergs/cm²-sec. In models RH-U2 and RH-M the modern values are in rough agreement with the observed heat flow. The heat flow in model RH-U2 has been lower than that in model RH-U6 since 2400 My ago. This results from the rapid core-mantle separation and the rapid near surface concentration of the radioactive heat sources at a time 4.5×10^9 years ago.

5. Discussion and Conclusions

In the process of the core formation, gravitational energy is transformed to thermal energy, to the amount of about 2800 joules/g on an average value for the whole mass of the Earth. This is equivalent to an average rise of temperature of about 2200° . The energy is higher than that given by Runcorn¹⁵⁾ and Birch³⁾ using different methods and is lower than that given by Urey¹³⁾, Stacey¹⁾, and Shimazu⁴⁾ using approximate methods. The large amount of energy released favors the view that the core was substantially

complete at the time between 4500 My and 3000 My ago, and that its process reached completion in a geologically short time. A rapid growth of the core is also confirmed by Stacey¹⁾, Tozer⁵⁾, and Shimazu^{4), 16)} by different thermodynamical calculations. For the moon, Iriyama and Shimazu¹⁷⁾ have obtained a similar conclusion.

The release of gravitational energy on core formation dominates the thermal state of the Earth. The molten state of the core is possible to attain even if the Earth is an initially cold and the core has no radioactive heat sources.

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