

Field Mapping of the Point-to-Plane Gap

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For the analysis of discharge in a point-to-plane gap (practically, one electrode is a hemispherical rod, the other being a circular disc), the gap is usually approximated by 1) confocal paraboloids of rotation, 2) confocal hyperboloids of rotation or 3) concentric spheres.

To examine which is the best among these approximations, the Poisson equation was solved by the method of difference equations, the boundary potentials being set equal to the measured voltages. In natural water in a large plastics box, the tip of an isolated thin wire was moved from point to point of the boundary by a travelling microscope and the voltages were read by a vacuum tube voltmeter with internal resistance 110M Ω , the applied voltage between the electrodes being 5V (A.C.) or so.

§ 1. Introduction

Both 1) the paraboloidal approximation and 2) the hyperboloidal approximation can be subdivisible to a) Approximation of coincidence of curvatures and b) Approximation of coincidence of 3 points, say. In the former the radius of curvature of the tip of the parabola (or hyperbola) is set equal to the radius of the hemisphere. In the latter the parabola (or hyperbola) passes the 3 points, the tip of the hemisphere and 2 end points of a diameter of the circle of intersection of the rod and the hemisphere.

The potentials on the shortest path between 2 electrodes calculated by the 5 approximations (involving the concentric spheres approximation) are compared with our calculations (the result of the difference equations).

§ 2. Formulae

Cases of l/r , where r is the radius of the hemisphere, l the gap length, $=160$ and $=20$ are calculated. By the former case, the steps of calculation will be explained. In

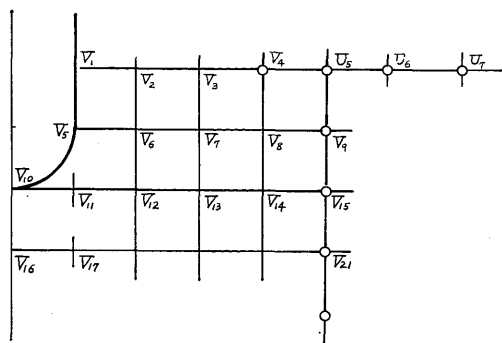


Fig. 1

the 2 dimensional space (Fig. 1) the lattice points are named $V_1, V_2, \dots$ and $U_5, U_6, \dots, U_9$ as shows in Fig. 1. The measurement of the votlages are done at the points $V_4, U_5, U_6, \dots, U_9, U_{15}, U_{21}, \dots$. Measurements at the points V_2, V_3 should be avoided in fear of dissurbance of the natural field. The potentials $V(2), V(3)$ at the point V_2, V_3 are interpolated graphically using the values $V(1), V(4), U(5), U(6), \dots, U(9)$.

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In the first step, $V(6)$, $V(7)$, $V(8)$ are set equal to $V(9)$, the values $V(11)$, $V(12)$, $V(14)$ equal to $V(15)$, and so on. At a point P , e.g. at V_{18} , owing to the following formula $W(P)$ is calculated.

$$\left\{ \frac{V(E)-W(P)}{h} - \frac{W(P)-W(W)}{h} \right\} \frac{1}{h} + \frac{1}{r} \frac{V(E)-W(W)}{2h} + \left\{ \frac{W(N)-W(P)}{h} - \frac{W(P)-V(S)}{h} \right\} \frac{1}{h} = 0$$

This is transformed to

$$W(P) = \left\{ V(E) + W(W) + W(N) + V(S) + \frac{V(E)-W(W)}{2n} \right\} / 4 \dots \dots \dots (1)$$

where $n=r/h$. E, W, N, S mean east, west, north and south respectively, and r is the distance of the point P from the axis of the gap, h being the lattice constant. At the point V_8 e.g., the formula

$$W(Q) = \left\{ V(E) + W(W) + V(N) + V(S) + \frac{V(E)-W(W)}{2n} \right\} / 4 \dots \dots \dots (1')$$

must be used, $W(N)$ being not used.

For the points on the axis, the term

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) \equiv D(r) \dots \dots \dots (1'')$$

involved in Poisson equation is approximated by

$$\left\{ \frac{V(E)-W(P)}{h} - \frac{h}{2} - 0 \right\} \frac{2}{h} \frac{4}{h} = \frac{4}{h^2} \{ V(E) - W(P) \}$$

This corresponds to $D(h/4)$ but not to $D(0)$. The difference equation is roughly assumed to be

$$\frac{4}{h^2} \{ V(E) - W(P) \} + \frac{1}{h^2} \{ W(N) + V(S) - 2W(P) \} = 0$$

This is transformed to

$$W(P) = \{ 4V(E) + W(N) + V(S) \} / 6 \dots \dots \dots (2)$$

Owing to these formulae, the potentials $W(P)$ at all lattice points except boundary points can be calculated rapidly by an electronic computer. Thus far is the first step, say.

The 2nd. step is as follows. By the formula in the electronic computer language

$$V(P) = V(P) + \{ W(P) - V(P) \} \times m \dots \dots \dots (3)$$

the new $V(P)$'s are computed, where m is so-called the accelerating factor ($1 \leq m < 2$), and we set $m=1.5$. With these new $V(P)$'s one goes again to equation (1) and repeats the same way.

Fig. 2 is the result of the calculation. The potentials were normalized throughout this work so as $V=1$ at the rod and $V=0$ at the plate.

To draw the equipotential contours near the hemispherical rode more correctly, the small portion of the gap shown in Fig. 3 was examined. Boundary values of the potentials are known from Fig. 2. Further formulae were found necessary for cases, as one of them is explained by Fig. 4

In Fig. 4 let $PN' = \alpha h$ ($0 < \alpha < 1$), then the difference equation at P is

$$\frac{V(E) + V(W) - 2W(P)}{h^2} + \frac{1}{2n h^2} \{ V(E) - V(W) \} + \left\{ \frac{V(N') - W(P)}{\alpha h} \right\}$$

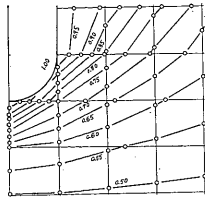


Fig. 2

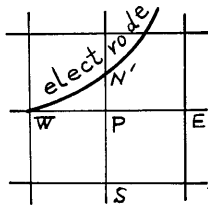


Fig. 4

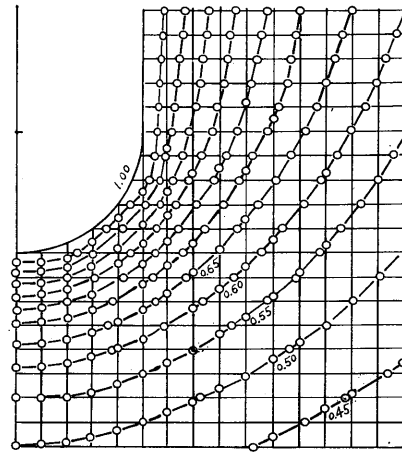


Fig. 3

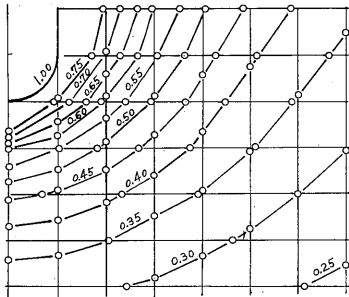


Fig. 5

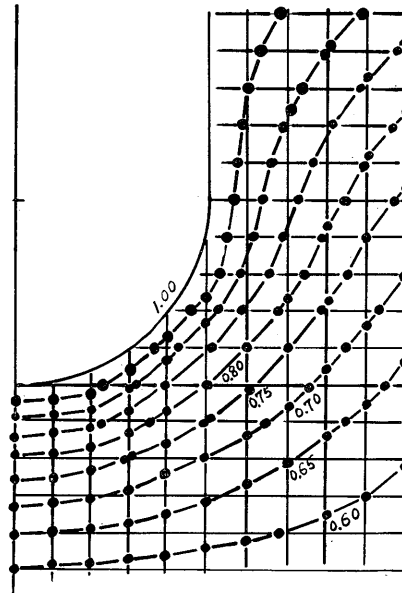


Fig. 6

$$-\frac{W(P)-V(S)}{h}\left\}/\left(\frac{1}{2}\alpha h+\frac{1}{2}h\right)=0$$

This is transformed to

$$W(P)=\{V(E)+V(W)+\{V(N')/\alpha+V(S)\}\times 2/(1+\alpha) \\ +\{V(E)-V(W)\}/2n\}/(2+2/\alpha) \dots\dots\dots (4)$$

The calculation similar to that done to get Fig. 2 gives Fig. 3. Figures 5 and 6 show cases for $l/r=20$

§ 3 Comparisons of 5 Approximations with Our Result and Conclusions

In Fig. 7 and 8, (A) shows our result, (B) paraboloidal, (C) hyperboloidal and (D) spherical approximation. If our work is justified, we can conclude that for case $l/r=160$ the paraboloidal approximation of 3 points coincidence and for case $l/r=20$ the

paraboloidal approximation of curvature coincidence seem to be the most adequate.

Next, though the field strength is strongest just outside the tip, there seem to exist also points of maximum strength a little distance apart from the tip along the hemisphere.

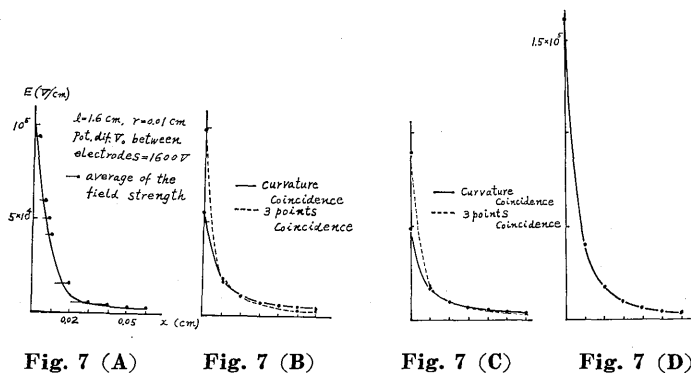


Fig. 7 (A)

Fig. 7 (B)

Fig. 7 (C)

Fig. 7 (D)

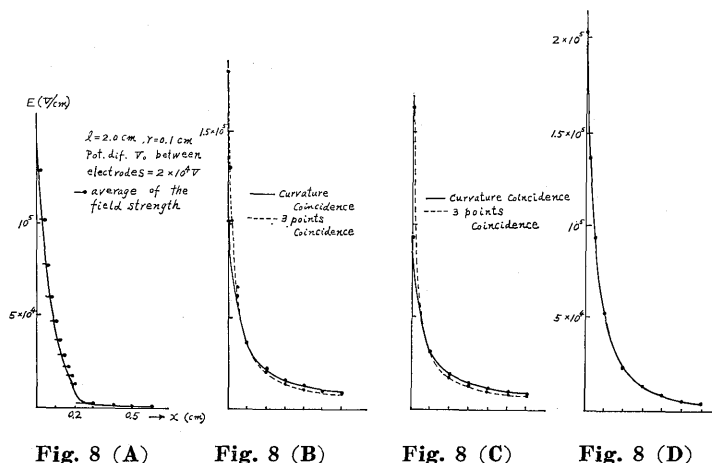


Fig. 8 (A)

Fig. 8 (B)

Fig. 8 (C)

Fig. 8 (D)

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